Universality in holography far from equilibrium

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Based on:
1912.00032 w/ T. Andrade, C. Pantelidou, J. Sonner
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Motivations

- We often study thermal equilibrium

• equilibrium

We understand how to characterise it macroscopically (thermodynamics) & its local neighbourhood (hydrodynamics / linear response…)

Holography as a microscopic model
  thermal equilibrium = Killing horizon in AdS
Motivations

- This talk is about the larger space:

  non-equilibrium landscape

  \[ \bullet \text{equilibrium} \]

  steady states

- Its larger because you can drive / prepare initial conditions in many ways

- **Holographic duality works in real-time:** arbitrary time-dependent spacetimes

- special case: non-equilibrium steady states
  
  **dual to** stationary non-Killing black branes
Motivations

• Goal: find special points exhibiting universality
• Want features insensitive to details of driving, initial conditions, models
• Focus on famous known example: turbulence
Motivations

\[
\left( \frac{\partial}{\partial t} - \nu_0 \frac{\partial^2}{\partial x^2} \right) u^i = M^i_{jk} u^j u^k
\]

• Turbulence relies on nonlinearities
• Observed widely in nature

• Interactions lead to cascades
• Power transferred between scales
• Information about initial conditions washed out
• Universal behaviour described by Kolmogorov (1941)
Motivations

Turbulence of strongly interacting QFTs?

• Focus on holographic QFTs with Einstein gravity duals

• Event horizons can behave like fluids, e.g.
  • Horizon behaves as viscous membrane (asy. flat space) [Damour] [Price, Thorne, MacDonald] (1986)
  • Shear viscosity of AdS black branes [Policastro, Son, Starinets 2001]
  • One-to-one map between near-equilibrium black holes and solutions to relativistic viscous hydro [Bhattacharyya et al. 2007]
  • Large D expansion [Emparan, Suzuki, Tanabe 2013] [Bhattacharyya, De, Minwalla, Mohan, Saha 2015]

• We may hope that turbulent universality is also seen
Outline

• Motivations
• Turbulence & Kolmogorov 1941
• Large D limit of GR
• Results
Turbulence

• Statistical distributions of velocity field $\vec{u}$
• here: homogeneous and isotropic turbulence (HIT)
  • pdf($\vec{u}$) translation & rotation invariant

Observables:
• n-pt functions of $\vec{u}$, averaged over realisations
• specialise to longitudinal ‘structure functions’

$$S_n = \langle |(\vec{u}(\vec{x} + \vec{y}) - \vec{u}(\vec{x})) \cdot \hat{y}|^n \rangle$$

• HIT $\Rightarrow S_n(|y|)$
• Instead of $S_2$ common to use velocity power spectrum

$$E(\vec{k}) = 4\pi k^2 \left\langle u_k u_{-\vec{k}} \right\rangle$$
• measures how much KE in shell radius $|k|$
Phenomenology (3+1)

\[ E \]

\[ k_d \sim \nu_0^{-3/4} \]

- stationary distribution

\[
\left( \frac{\partial}{\partial t} - \nu_0 \frac{\partial^2}{\partial x^2} \right) u^i = M^i_{jk} u^j u^k
\]

Energy transfer
Phenomenology (2+1)

- Quasi-stationary (up to IR effects)
- Stationarity typically achieved with a friction term
Kolmogorov 1941 (‘K41’)

Similarity hypotheses
1. pdfs determined uniquely by $\nu_0$, $\varepsilon$
   $\varepsilon$ : energy transfer rate
2. viscous effects only influence high k
   i.e. $\exists$ an ‘inertial range’ where pdfs depend only on $\varepsilon$

Dimensional analysis

$$
[\varepsilon] = \frac{[v]^2}{[t]} = \frac{[\ell]^2}{[t]^3} \quad [S_2] = \frac{[\ell^2]}{[t^2]}
$$

$$
\Rightarrow S_2(r) = C\varepsilon^{2/3}r^{2/3} \quad \text{‘two-thirds law’}
$$

or,

$$
E(k) = C\varepsilon^{2/3}k^{-5/3}
$$

K41 also did $S_3$
commonly stated:

$$
S_n(r) = C\varepsilon^{n/3}r^{n/3}
$$

(Independent of number of dimensions)
K41 in holography

- Directly solve Einstein equations numerically?
  - no symmetries
  - no characteristic scales
  - black box problem

- Want analytic control

- GR has a dimensionless parameter: D
  - perturbation theory in 1/D
  - its a good idea for black holes (two clues)
  - separation of scales $r_0 \gg r_0/D$
  - construct a EFT

- Better control than hydrodynamics
Large D limit of GR. Clue 1: localisation of potential

Schwarzschild-AdS\(_{d+1}\)

\[ ds^2 = -r^2 f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 d\mathbf{x}^2 \]

\[ f(r) = 1 - \left( \frac{r_0}{r} \right)^d \]

Take \( d \to \infty \) with \( r_0 \) fixed

\[ f(r) \to 1 \]

Vacuum AdS, corrections,

\[ f(r) \to 1 + O(e^{-\frac{d(r-r_0)}{r_0}}) \]

Potential localised to thin strip near \( \mathcal{H} \)

\[ \frac{r_0}{d} \]

Radial gradients stronger than transverse gradients
Large D limit of GR. Clue 2: QNM spectrum

\[ \delta \phi = \phi_{nk}(r) e^{ikx - i\omega_n t} \]

Decoupling between near-horizon and far zones

Spectrum:

\[ \omega_n = O(d)^1 \] \quad \text{‘heavy’} \quad \text{decoupled to all perturbative orders in } 1/d

\[ \omega_n = O(d)^0 \] \quad \text{‘light’}
Construction of large D EFT

1. Take boosted Schwarzschild metric $g_0(r)$, depends on $a, p^i$

2. Promote moduli to functions of boundary coords
   
   $a, p^i \rightarrow a(t, \vec{x}), p^i(t, \vec{x})$

3. $g_0(t, r, \vec{x})$ doesn’t solve Einstein equations but,
   
   \[ \text{error} \sim 1/d \]

4. Correct metric
   
   $g_0(t, r, \vec{x}) + \frac{1}{d} g_1(t, r, \vec{x})$

5. Can analytically solve radial \textit{evolution equations}
   
   \[ \text{error} \sim 1/d^2 \]

6. And so on.

Only radial \textit{constraint equations} remain
Construction of large D EFT

The radial constraints are,

\[
\begin{align*}
(\partial_t - \nabla^2) a &= -\nabla \cdot p \\
(\partial_t - \nabla^2) p^i &= -\nabla^i a - \nabla_j \left( \frac{p^i p^j}{a} \right)
\end{align*}
\]

Comments:

- Depend only on the horizon/boundary directions \( t, x^i \)
- Once solved, we have a full \( d + 1 \) black hole metric
- Looks like non-relativistic hydro, but exact in gradients

Next: solve numerically and check K41
(restrict to 2+1, work on a torus, ask me for details)
unstable initial data

(movie)
Homogeneous Isotropic Driving

- Want quasi-stationary turbulence, need to drive.
- Follow traditions of NSE community,

\[
\begin{align*}
(\partial_t - \nabla^2) a &= -\nabla \cdot p \\
(\partial_t - \nabla^2) p^i &= -\nabla^i a - \nabla_j \left( \frac{p^i p^j}{a} \right) + F_i
\end{align*}
\]

- \(F_i\) : is a Wiener process that injects vorticity
  - isotropic sum of k-modes with random amplitudes and phases
- Start from thermal equilibrium
$F_i$ (Forcing)

(movie)
- Power Spectrum (256 realisations)

\[ k^\frac{-5}{3} \]

\[ \log E \]

\[ kL/(2\pi) \]

vortex lattice

\[ t = 24L \]

\[ t = 12L \]

\[ t = 6L \]

\[ t = 2L \]

driving
- Structure functions (256 realisations)

\[ S_n \propto r^{\zeta_n}, \quad \zeta_n = \frac{n}{3} \]
• $F_i$ natural from a fluids community perspective

• We can also deform the CFT metric, e.g.

$$g_{tt}(t, \vec{x}) = -1 + \frac{\gamma_{tt}(t, \vec{x})}{d}$$

changes BVP in AdS

• Same equations with [Andrade, Pantelidou, BW 2018]

$$F_i = \frac{a}{2} \nabla_i \gamma_{tt}$$

• But doesn’t directly inject vorticity!
• Regardless, we can find turbulent flows
• Quench a cylindrical $\gamma_{tt}$ potential
Quenching

\( \gamma_{tt} \)

(movie)
Summary & Outlook

• Sought universal dynamics in far from equilibrium strongly interacting matter

• Demonstrated K41 for black holes in AdS

• Microscopic model, no hydrodynamic limit used

• $d \to \infty$
  • Hydrodynamic limit: fluid-gravity duality works at any $d$
  • K41 dimensional analysis is dimension independent
  • Proof of principle of universal dynamics far from equilibrium

• Turbulent wakes for dragged obstacle
  • Instability of NESS?

• Turbulence:
  • Large-$d$ effective equations
  • Similar to NSE but no gradient expansion
  • Gradient-exact toy model with parametric control ($1/d$)

Thank you!