SURPRISES IN HOLOGRAPHIC QUANTUM CRITICAL TRANSPORT

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based on work with Simon Gentle & Blaise Goutéraux

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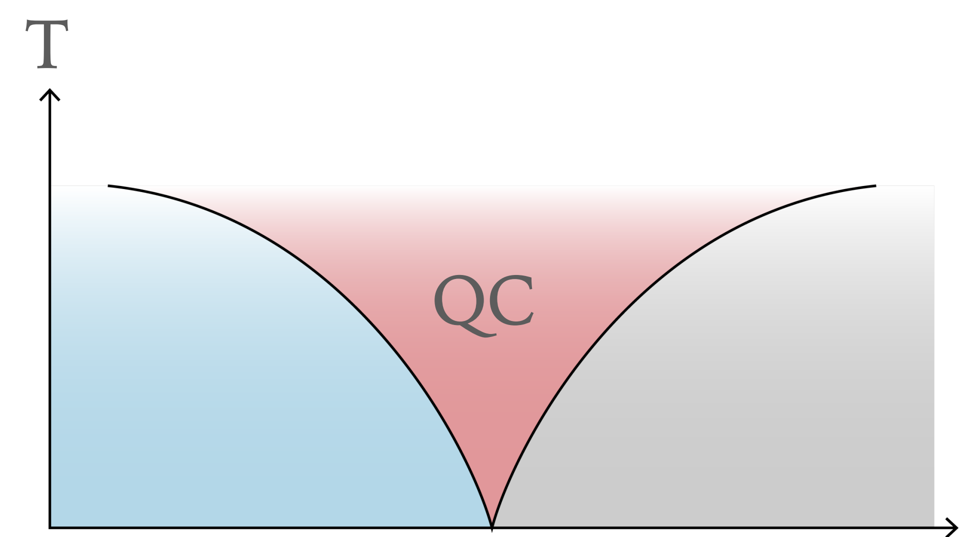
MOTIVATION: QUANTUM CRITICALITY

- Quantum critical states of matter arise near quantum critical points

- Exhibit robust scaling behavior of observables

  e.g. $\xi \sim T^{-1/z}$ z: dynamical critical exponent

  $\tau = T^{-1}$



- Scaling behavior observed in the cuprates (and other strange metals) makes them prime candidates.
  - ✳ Simple scaling theories are unsuccessful Phillips & Chamon (2005)
  - ✳ More sophisticated attempts are less unsuccessful Hartnoll & Karch (2015)

HOLOGRAPHIC QUANTUM CRITICALITY

- I will discuss very simple holographic examples of quantum critical states:
    - translational invariance
    - unbroken U(1) symmetry with $\langle Q \rangle \neq 0$

- Their properties seem to be inconsistent even with 'sophisticated' scaling theories à la Hartnoll & Karch.

RD, Goutéraux and Hartnoll (2015)

- The first part of my talk: what is the correct scaling theory?

  i.e. what are the scaling dimensions of thermodynamic and transport observables in these phases?

HOLOGRAPHIC QUANTUM CRITICALITY

- 2nd part: how is the scaling theory manifested in holographic examples?

  There is a surprisingly rich set of possible quantum critical dynamics

  $* \, z \neq 1$: T is the only scale
  T-scaling of follows from dimensional analysis e.g. $\tau \sim 1/T$

  $* \, z = 1$: some observables controlled by a dangerously irrelevant deformation of the critical point

  ⟶ non-Planckian relaxation and transport

- In less symmetric (i.e. more realistic) examples, there is likely a much richer set of possibilities.

SCALING OF THERMODYNAMIC QUANTITIES

- Assume there is a symmetry associated to rescaling space and time. Scaling dimensions tell us how objects transform under this symmetry.

- Assign units such that $[x] = -1$ and $[t] = -z$ $\longrightarrow$ $[T] = z$

  i.e. z is the dynamical critical exponent: $\xi = T^{-1/z}$

- Assume that the entropy density s and charge density ρ transform nicely. Naively:

$$[s] = [\rho] = d$$

d: spatial dimensionality

- We will allow each object to have an anomalous dimension

$$[s] = d - \theta \qquad [\rho] = d - \theta + \Phi$$

Charmousis et al (2010), ….

RESPONSE FUNCTIONS AND TRANSPORT

- Which response functions are controlled by the quantum critical dynamics?

  Those will exhibit scaling.

- At long times & distances, local conservation of entropy density δs, charge density $\delta\rho$ and momentum density $\delta\pi$ control the dynamics.

- The thermoelectric conductivities are (χ = static susceptibility)

$$\sigma(\omega) = \frac{\rho^2}{\chi_{\pi\pi}}\frac{i}{\omega} + \sigma_0 + O(\omega), \qquad \alpha(\omega) = \frac{s\rho}{\chi_{\pi\pi}}\frac{i}{\omega} + \alpha_0 + O(\omega), \qquad \bar{\kappa} = \frac{Ts^2}{\chi_{\pi\pi}}\frac{i}{\omega} + \bar{\kappa}_0 + O(\omega)$$

  The divergent pieces at small ω are due to the drag of (conserved) momentum.

RESPONSE FUNCTIONS AND TRANSPORT

- We can nicely isolate the response that is independent of momentum drag by changing variables $(\delta\rho, \delta s) \longrightarrow (\delta p, \delta\rho_{inc})$ where

$$\delta p \equiv s\delta T + \rho\delta\mu \qquad\qquad \delta\rho_{inc} \equiv s^2 T\delta(\rho/s)$$

  $\delta\rho_{inc}$ has a finite d.c. conductivity as its current j_{inc} does not drag momentum.

- δp propagates ballistically via sound waves while $\delta\rho_{inc}$ diffuses incoherently:

$$G^R_{\rho_{inc}\rho_{inc}}(\omega, k) = \frac{k^2\sigma^{dc}_{inc}}{i\omega - Dk^2} \qquad\qquad D = \sigma^{dc}_{inc}\chi^{-1}_{inc}$$

- In terms of the s and ρ response functions:

$$\sigma^{dc}_{inc} = T^2 s^2 \sigma_0 - 2\rho s T^2 \alpha_0 + \rho^2 T \bar{\kappa}_0, \qquad \chi_{inc} = T^2\left(\rho^2\chi_{ss} - 2s\rho\chi_{s\rho} + s^2\chi_{\rho\rho}\right)$$

SCALING OF RESPONSE FUNCTIONS

- We will **only** assign scaling dimensions to $\delta\rho_{inc}$
- Pressure perturbations δp are sensitive to UV (non-quantum critical) physics arising from the exact conservation of momentum.

$\longrightarrow$ don't expect nice transformations under space and time rescalings

- This means that we do not expect objects like $\sigma(\omega)$ or $\chi_{\rho\rho}$ to exhibit quantum critical scaling.

  Instead we will deal directly with the objects that do: $\sigma_{inc}(\omega)$ and χ_{inc}.

- It's more convenient to use σ_{inc} and D (χ_{inc} follows from the Einstein relation)

SCALING OF RESPONSE FUNCTIONS

- From its definition $\delta\rho_{inc} = s^2 T\delta(\rho/s)$ $\longrightarrow$ $[\rho_{inc}] = 2(d-\theta) + z + \Phi$

  The dimension of the free energy is $[sT] = d - \theta + z$

  $\longrightarrow$ the dimension of the source of $\delta\rho_{inc}$ is $[s_{inc}] = -(d-\theta) - \Phi$

- The current is defined by the continuity $[j_{inc}] = 2(d-\theta) + 2z - 1 + \Phi$

  $\longrightarrow$ $[j_{inc}] = 2(d-\theta) + 2z - 1 + \Phi$ and $[E_{inc}] = -(d-\theta) - \Phi + 1$

- The charge and current response functions are ratios of these

  $[\sigma_{inc}] = 3(d-\theta) + 2(z - 1 + \Phi)$ and $[\chi_{inc}] = 3(d-\theta) + z + 2\Phi$

  and thus $[D] = z - 2 = 2[x] - [t]$

SCALING THEORIES

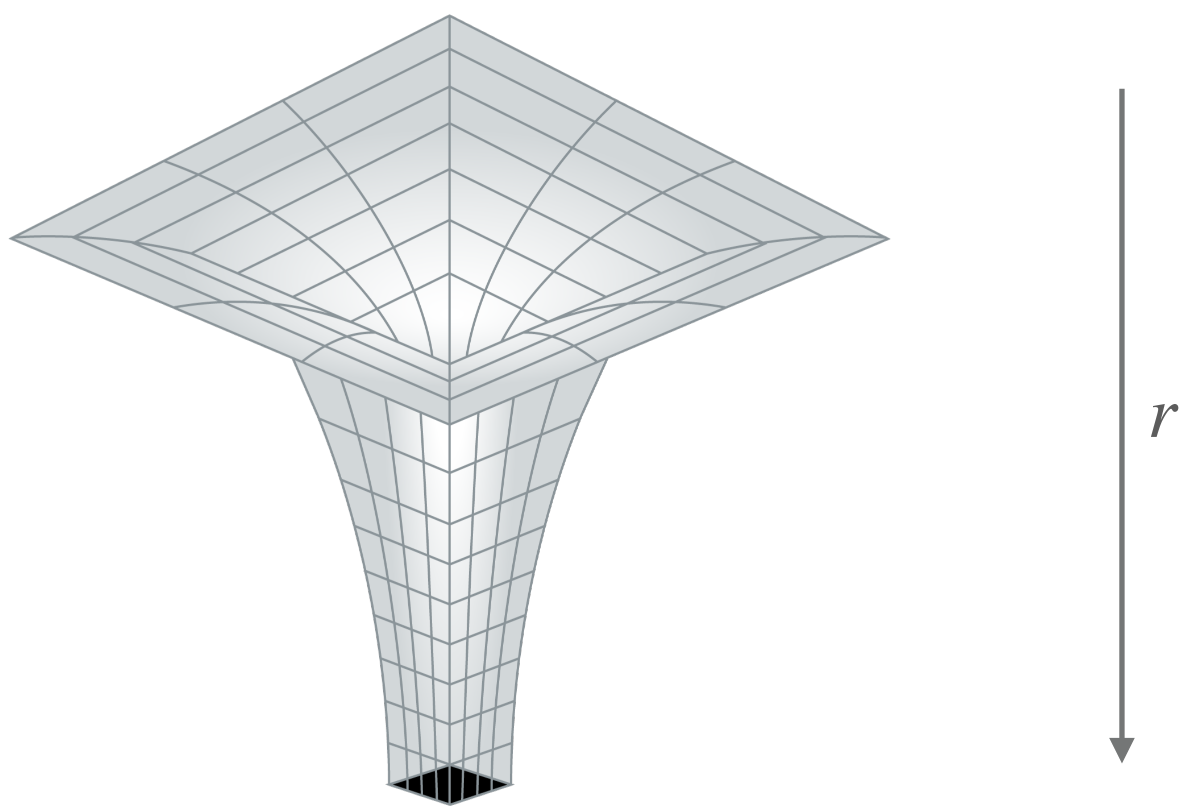
- Scaling theories are useful because they tell us the parametric dependence of objects on energy scales, independent of the microscopic details.

- If T is the only dimensionful scale, we can convert scaling dimensions to power law T-dependence using $[T] = z$:
  - $[t] = -z \longrightarrow \tau \sim T^{-1}$
  - $[\sigma_{inc}] = 3(d-\theta) + 2(z-1+\Phi) \longrightarrow \sigma_{inc}^{dc} \sim T^{\big(3(d-\theta)+2(z-1+\Phi)\big)/z}$
  - $[D] = z - 2 \longrightarrow D \sim T^{1-2/z}$

- Next: show that holographic examples are consistent with the scaling theory. But with a much richer variety of transport properties than written above.

HOLOGRAPHIC QUANTUM CRITICAL PHASES

- Gravity description: charged black hole of Einstein-Maxwell-dilaton theory



$$ds^2 = \frac{-dt^2 + d\underline{x}_d^2 + dr^2}{r^2}$$

$$ds^2 = \left(\frac{r}{L}\right)^{2\theta/d} \left(-\frac{L^{2z}}{r^{2z}} L_t^2 dt^2 + \frac{\tilde{L}^2 dr^2}{r^2} + \frac{L^2}{r^2} L_x^2 d\underline{x}_d^2 \right)$$

- At $T = 0$, the metric deep in the interior has a scaling symmetry

$$(r, \underline{x}) \to \Lambda(r, \underline{x}) \quad \text{and} \quad t \to \Lambda^z t$$

This is the quantum critical scaling, with $[r] = -1$.

HOLOGRAPHIC QUANTUM CRITICAL PHASES

- Different gravity theories give rise to different critical points.

  Classify them using the action governing the dynamics near the scaling region

$$S_{crit} = \int d^{d+2}x\sqrt{-g}\left(R - \frac{1}{2}\left(\partial\phi\right)^2 - V_0 e^{-\delta\phi} + Z_0 e^{\gamma\phi}F^2\right)$$

  The final term is a deformation away from the critical point by coupling g.

  Charmousis et al (2010), ...

- There are two classes of solution, dependent on the value of γ (the dimension Δ_g of g):

  * marginal deformation $\Delta_g = 0$, broken Lorentz and charge conjugation symmetry $\longrightarrow z \neq 1$

  * irrelevant deformation $\Delta_g < 0 \longrightarrow z = 1$

HOLOGRAPHIC SCALING PROPERTIES

- For small $T \neq 0$, the scaling properties are inherited by the metric near the black hole horizon.

  It is clear that only observables that depend just on the physics near the horizon will exhibit quantum critical scaling.

- The temperature and entropy of the black hole are set by the horizon radius r_h

$$T = \frac{(d+z-\theta)\,L_t}{4\pi\tilde{L}}\left(\frac{r_h}{L}\right)^{-z} \quad \text{and} \quad s = 4\pi L_x^d \left(\frac{r_h}{L}\right)^{\theta-d} \quad \longrightarrow s \sim T^{(d-\theta)/z}$$

- More abstractly, in terms of scaling dimensions (using $[r] = -1$):

$$[T] = z \qquad\qquad [s] = d - \theta$$

HOLOGRAPHIC SCALING PROPERTIES

- The black hole charge is independent of r_h but depends on the coupling g :

$$\rho = L_x^d \frac{\left(\theta - d - z - 2\Delta_g\right) Z_0}{\tilde{L}} g \qquad \longrightarrow \rho \sim T^0$$

  Its scaling dimension is $[\rho] = \Delta_g$.

- In other words, Δ_g sets the anomalous dimension Φ of the scaling theory

$$[\rho] = \Delta_g = d - \theta + \Phi$$

  The two different classes of critical point correspond to different anomalous dimensions for ρ.

HOLOGRAPHIC TRANSPORT PROPERTIES

- Now we could just use the scaling theory to get the transport properties:

$$[\sigma_{inc}] = d - \theta + 2(z - 1 + \Delta_g) \qquad \text{and} \qquad [D] = z - 2$$

Assuming the only dimensionful scale they depend on is T:

$$\sigma_{inc}^{dc} \sim T^{(d-\theta+2(z-1+\Delta_g))/z} \qquad \text{and} \qquad D \sim T^{1-2/z}$$

- But this does not agree with direct calculations in holographic examples!

$$\sigma_{inc}^{dc} \sim T^{(d-\theta+2(z-1+\Delta_g))/z} \qquad \text{and} \qquad D \sim g^{-2}T^{1-2/z+2\Delta_g/z}$$

When g is irrelevant, diffusivity is parametrically larger than expected.

- Naive T-scaling fails as D is controlled by the irrelevant coupling g.

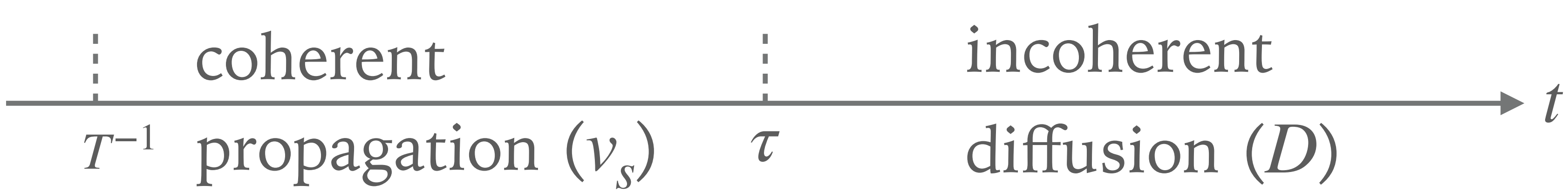
AN ANALOGY: MOMENTUM RELAXATION

- The irrelevant coupling plays a crucial role in the transport and relaxation properties of the system.

- **Analogy**: QCP where translational symmetry broken by irrelevant coupling g.

∗ Momentum relaxes over a parametrically long timescale $\tau \gg T^{-1}$ that depends on g.

∗ New IR timescale $\tau \longrightarrow$ much richer low energy transport properties



$$D = v_s^2 \tau$$

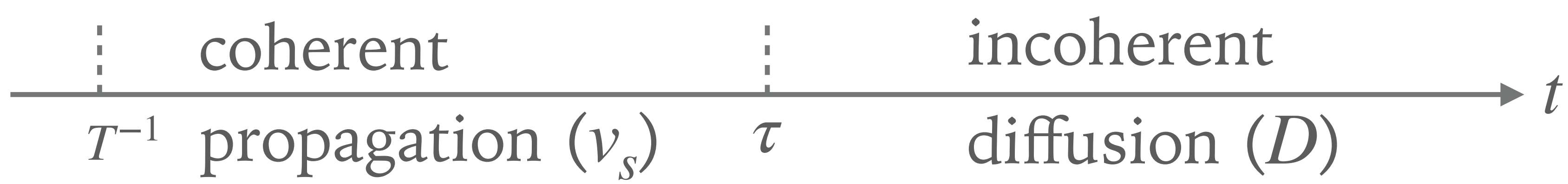
∗ D is parametrically larger than naively expected, as it is set by τ and not T^{-1}

SLOW RELAXATION AND DIFFUSIVITY

- In our case, the irrelevant coupling g controls the parametrically slow relaxation of j_{inc} :

$$\tau \sim \frac{1}{T}\frac{T^{2\Delta_g}}{g^2} \gg \frac{1}{T} \qquad\qquad \sigma_{inc}(\omega) = \frac{\sigma_{inc}^{dc}}{1 - i\omega\tau}$$

- We propose that, as a consequence, the low energy transport of $\delta\rho_{inc}$ undergoes a crossover to coherent propagation



⟶ D is anomalously large because it is set by τ : $D = v_s^2\tau$

- We are working to confirm this theory of transport quantitatively.

SUMMARY

- Studied system with a conserved momentum and non-zero density.

  Proposed a scaling theory for the sector of thermodynamics and transport that is independent of momentum drag.

- The scaling theory is consistent with all holographic examples.

  Surprisingly, the relaxation time and diffusivity are set by a coupling g which breaks Lorentz and particle-hole symmetries.

- When g is irrelevant, relaxation and diffusion are parametrically slow:

$$\tau \gg 1/T \qquad \text{and} \qquad D \gg T^{1-2/z}$$

  and we expect a much richer spectrum of low energy transport.

FURTHER WORK

- The most fundamental question

  Why does the relaxation timescale of j_{inc} depend on the irrelevant coupling g?

- Going beyond just these very symmetric examples:

  * Spontaneous translational symmetry breaking: can j_{inc} have similarly rich dynamics?

  * Explicit translational symmetry breaking: are there two independent transport timescales arising from independent relaxation of j_{inc} and π ?

  * Can we have slow relaxation of j_{inc} at QCPs with $z \neq 1$?

  * Magnetotransport

THANK YOU!