Quantum quenches and Fano resonances

in holographic strange metals with magnetic impurities

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Kondo effect:

Screening of a magnetic impurity by conduction electrons at low temperatures

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Motivation for study within gauge/gravity duality:

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Screening of a magnetic impurity by conduction electrons at low temperatures

Motivation for study within gauge/gravity duality:

1. New applications of gauge/gravity duality to condensed matter physics:
   - Magnetic impurity coupled to strongly correlated electron system
   - Entanglement entropy, quantum quenches

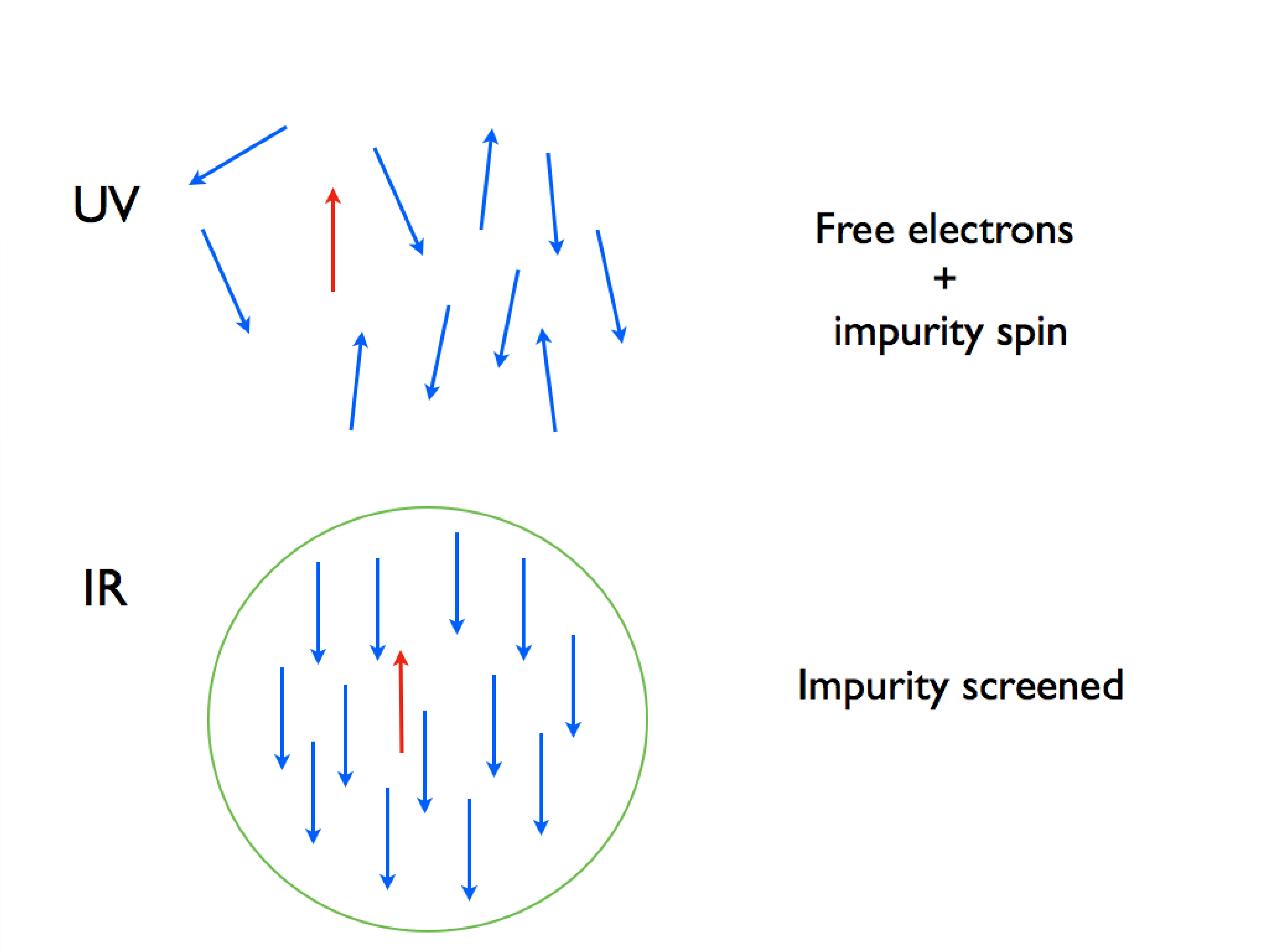
2. Simple model for a RG flow with dynamical scale generation (as in QCD)

3. Example for holographic g-theorem

4. Relation to Sachdev-Ye-Kitaev model

- – Model J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086
  - – Entanglement entropy J.E., Flory, Newrzella 1410.7811, JHEP 1501 (2015) 058 J.E., Flory, Hoyos, Newrzella, O'Bannon, Wu 1511.03666, Fortsch.Phys. 64 (2016)
  - – Two-point functions J.E., Hoyos, O'Bannon, Papadimitriou, Probst, Wu JHEP 1703 (2017) 039 , PRD 96 (2017) no.2, 021901
  - – Quantum quenches J.E., Flory, Newrzella, Wu JHEP 1704 (2017) 045

Kondo effect



Kondo model

Original Kondo model (Kondo 1964):
Magnetic impurity interacting with free electron gas

Hamiltonian:

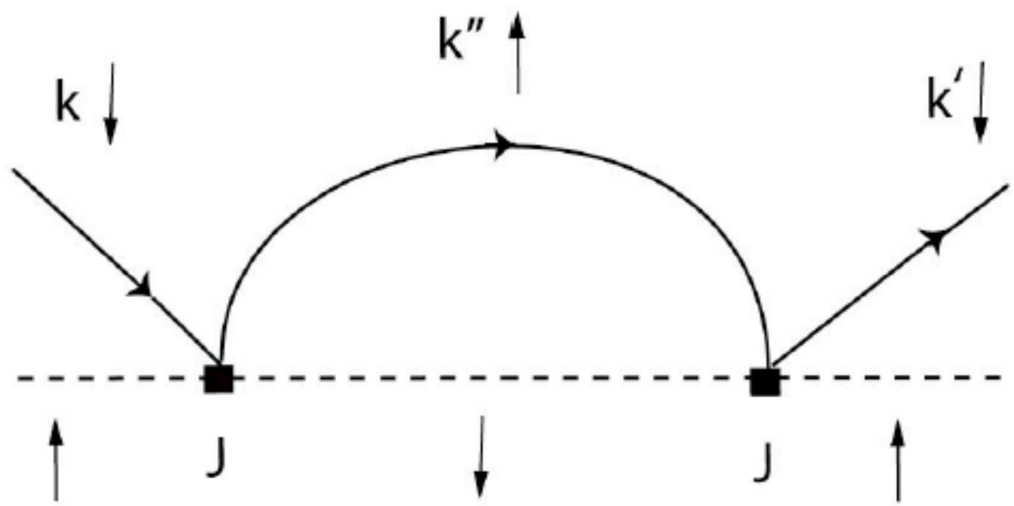
$$H = \frac{v_F}{2\pi} \psi^\dagger i \partial_x \psi + \lambda_K v_F \delta(x) \vec{S} \cdot \vec{J}, \quad \vec{J} = \psi^\dagger \frac{1}{2} \vec{T} \psi$$

Decisive in development of renormalization group
IR fixed point, CFT approach Affleck, Ludwig '90's

Solution using Bethe ansatz Andrei, WIegmann 80's

Large N Kondo model Read, Newns, Coleman, ... 80's

Scattering with magnetic impurities



$$\rho \sim \rho_0 \left(1 + \kappa \log \frac{T}{|\epsilon - \epsilon_F|} \right)$$

Antiferromagnetic coupling $\kappa < 0$

Logarithmic behaviour at low temperatures

J. Kondo 1964

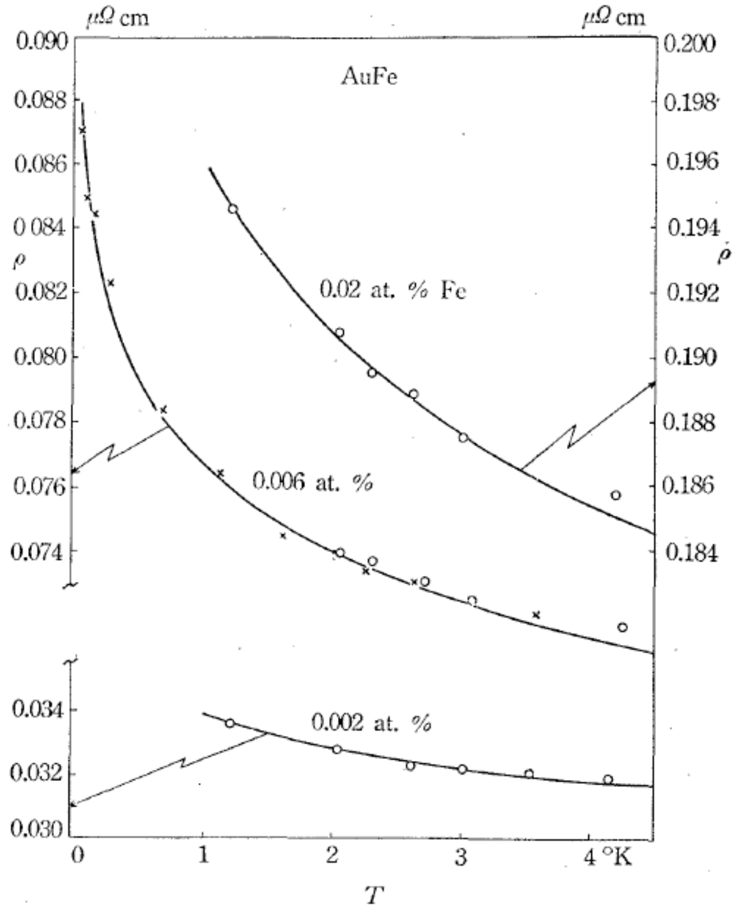


Fig. 1. Comparison of experimental and theoretical ρ–T curves for dilute AuFe alloys.

Breakdown of perturbation theory

$$\rho \sim \rho_0 \left(1 + \kappa \log \frac{T}{|\epsilon - \epsilon_F|}\right)$$

Perturbation theory breaks down at $T_K = |\epsilon - \epsilon_F| e^{1/\kappa}$

T_K: Kondo temperature

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$T_K \sim \Lambda_{\text{QCD}}$

Gauge/gravity requires large N: Spin group $SU(N)$

Kondo models from gauge/gravity duality

Gauge/gravity requires large N: Spin group $SU(N)$

In this case, interaction term simplifies introducing slave fermions:

$S^a = \chi^\dagger T^a \chi$

Totally antisymmetric representation: Young tableau with Q boxes

Constraint: $\chi^\dagger \chi = Q$

Interaction: $J^a S^a = (\psi^\dagger T^a \psi)(\chi^\dagger T^a \chi) = \mathcal{O}\mathcal{O}^\dagger$, where $\mathcal{O} = \psi^\dagger \chi$

Screened phase has condensate $\langle \mathcal{O} \rangle$

Coleman PRB 35, 5072 (1987)

Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192, PRB 58 (1998) 3794

Senthil, Sachdev, Vojta cond-mat/0209144, PRL 90 (2003) 216403

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Coupling of a magnetic impurity to a strongly interacting non-Fermi liquid

Kondo models from gauge/gravity duality

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Coupling of a magnetic impurity to a strongly interacting non-Fermi liquid

Results:

- RG flow from perturbation by 'double-trace' operator
- Dynamical scale generation, screening
- Holographic superconductor: Condensate forms in AdS_2
- Power-law scaling of resistivity in IR with real exponent
- Holographic entanglement entropy from including backreaction
- Quantum quench: Equilibration dominated by quasinormal modes
- Fano resonance in spectral function (spectral asymmetry)

Kondo models from gauge/gravity duality

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

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Top-down brane realization

	0	1	2	3	4	5	6	7	8	9
N D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

- 3-7 strings: Chiral fermions ψ in 1+1 dimensions
- 3-5 strings: Slave fermions χ in 0+1 dimensions
- 5-7 strings: Scalar (tachyon)

D3: $AdS_5 \times S^5$

D7: $AdS_3 \times S^5 \rightarrow$ Chern-Simons A_μ dual to $J^\mu = \psi^\dagger \sigma^\mu \psi$

D5: $AdS_2 \times S^4 \rightarrow \begin{cases} \text{YM } a_t \text{ dual to } \chi^\dagger \chi = q \\ \text{Scalar dual to } \psi^\dagger \chi \end{cases}$

Operator		Gravity field
Electron current J	$\Leftrightarrow$	Chern-Simons gauge field A in AdS_3
Charge $Q = \chi^\dagger \chi$	$\Leftrightarrow$	2d gauge field a in AdS_2
Operator $\mathcal{O} = \psi^\dagger \chi$	$\Leftrightarrow$	2d complex scalar Φ

Action:

$$S = S_{\rm Einstein-Hilbert} + S_{CS} + S_{AdS_2},$$

$$S_{CS} = -\frac{N}{4\pi} \int_{AdS_3} {\rm Tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right),$$

$$S_{AdS_2} = -N \int dx dt dz \, \delta(x) \sqrt{-g} \left[\frac{1}{4} {\rm Tr} f^{mn} f_{mn} + g^{mn} \left(D_m \Phi \right)^\dagger D_n \Phi + V(\Phi^\dagger \Phi) \right]$$

$$V(\Phi) = M^2 \Phi^\dagger \Phi$$

Metric: BTZ black hole

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{1}{z^2} \left(\frac{dz^2}{h(z)} - h(z) \, dt^2 + dx^2 \right),$$

$$h(z) = 1 - z^2/z_H^2 \,, \qquad T = 1/(2\pi z_H)$$

AdS_2 gauge field: asymptotically $a_t = \frac{Q/N}{z} + \mu$

‘Double-trace’ deformation by $\mathcal{O}\mathcal{O}^\dagger$

Boundary expansion

$$\Phi = z^{1/2}(\alpha \ln z + \beta)$$

$$\alpha = \kappa\beta$$

κ dual to double-trace deformation

Witten hep-th/0112258

Berkooz, Sever, Shomer hep-th/0112264

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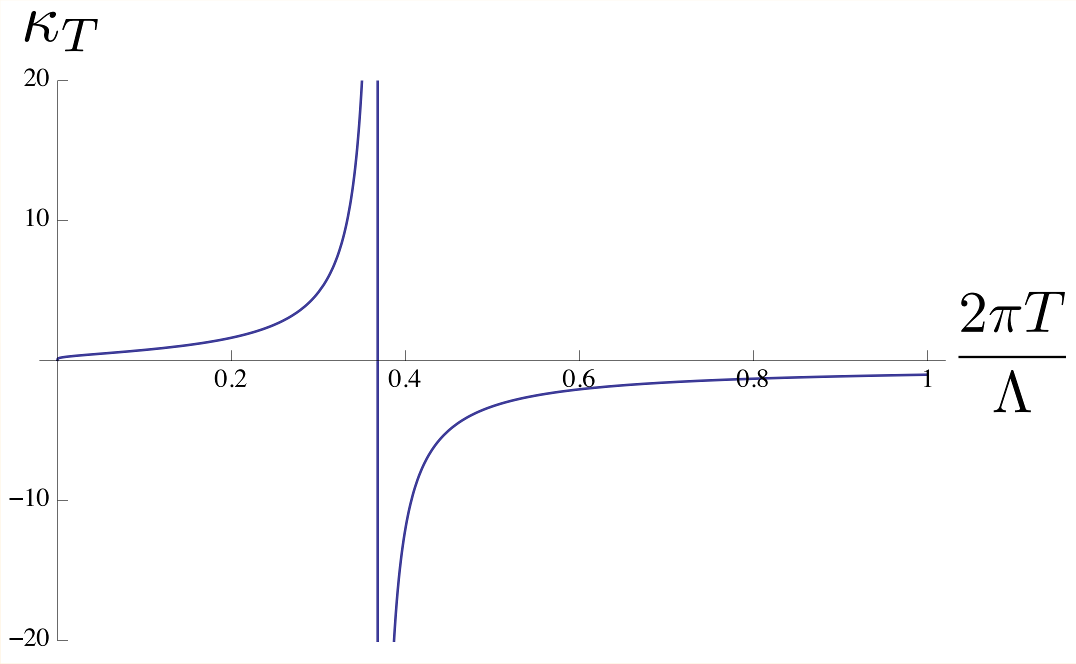
Φ invariant under renormalization $\Rightarrow$ Running coupling

$$\kappa_T = \frac{\kappa_0}{1 + \kappa_0 \ln\left(\frac{\Lambda}{2\pi T}\right)}$$

Dynamical scale generation

Scale generation

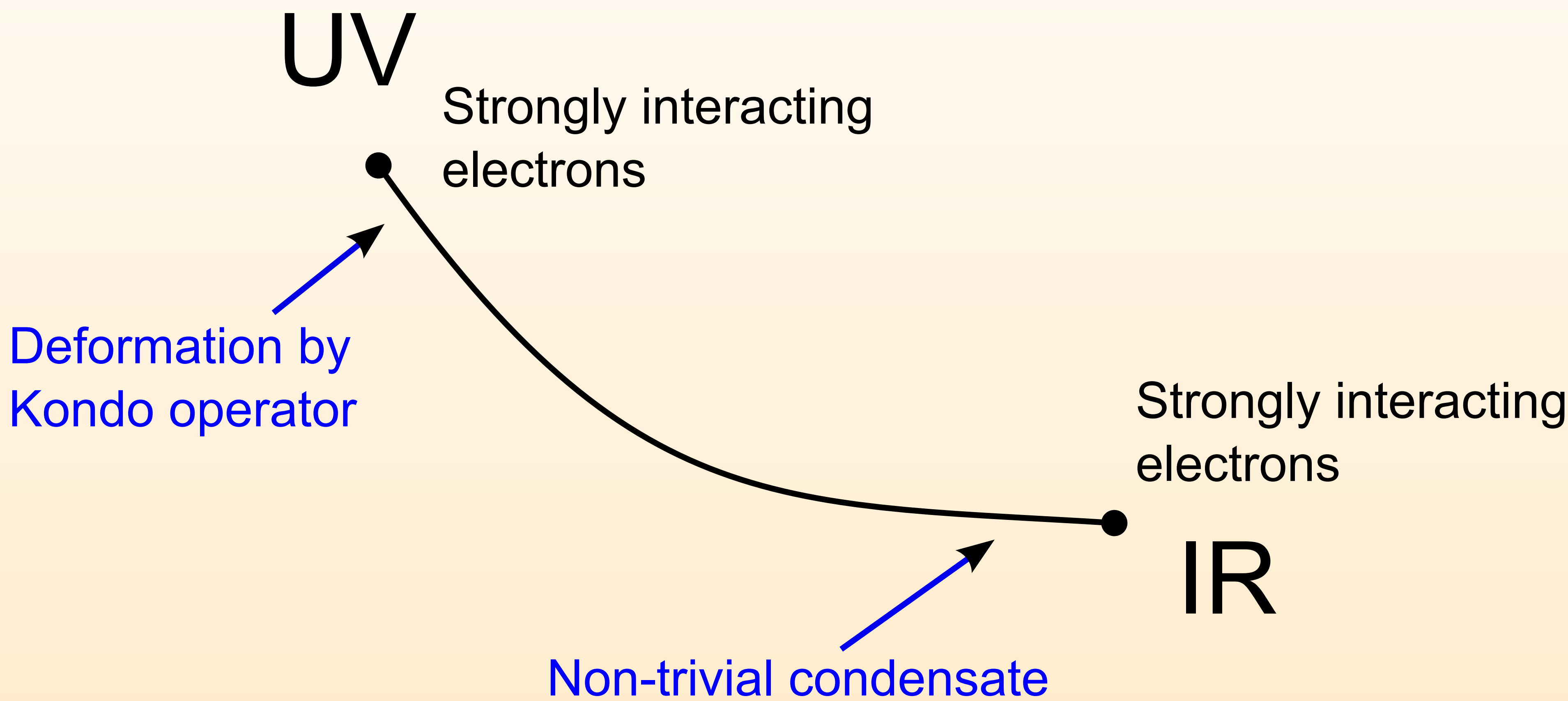


Divergence of Kondo coupling determines Kondo temperature T_K

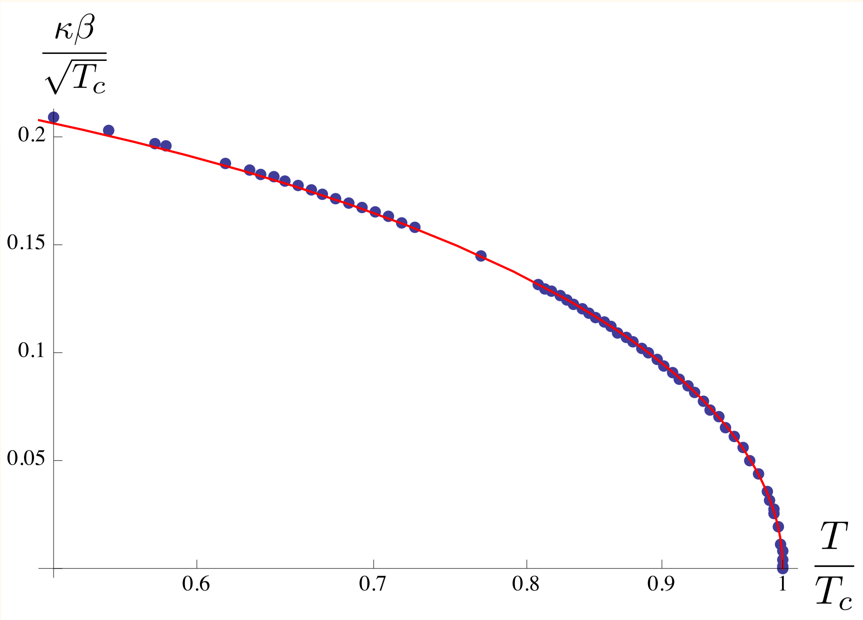
Transition temperature to phase with condensed scalar: T_c

$T_c < T_K$

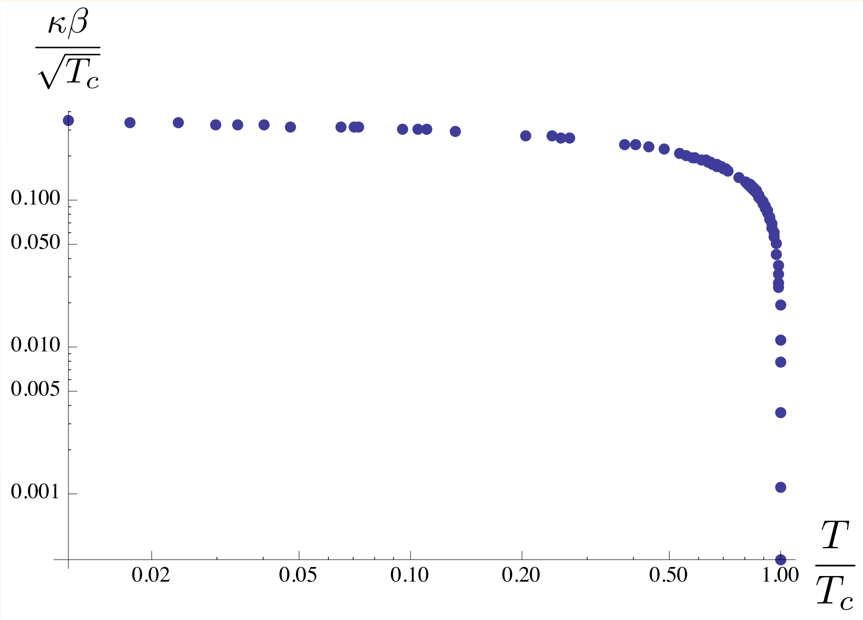
RG flow


UV
Strongly interacting
electrons
Deformation by
Kondo operator
Strongly interacting
electrons
IR
Non-trivial condensate

Normalized condensate $\langle \mathcal{O} \rangle \equiv \kappa\beta$ as function of the temperature



(a)

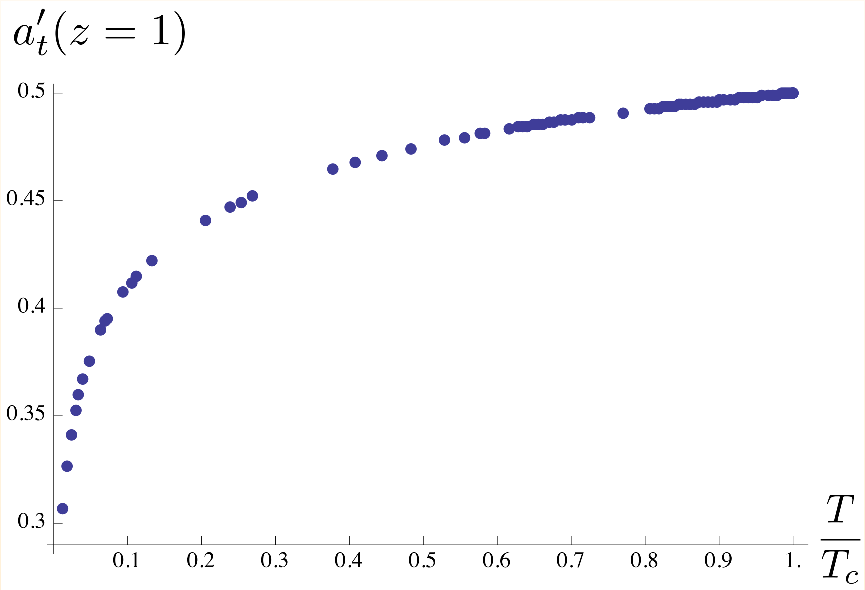


(b)

Mean field transition

$\langle \mathcal{O} \rangle$ approaches constant for $T \to 0$

Electric flux at horizon



(a)

$$\sqrt{-g}f^{tr}\Big|_{\partial AdS_2} = q$$

charge $q = Q/N$ of 2d gauge field determines impurity representation

Impurity is screened

Quantum quench and time dependence

Allow for time dependence of the Kondo coupling and study response of the condensate

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Examples for time dependence of the Kondo coupling:

- Gaussian pulse in IR
- Quench from condensed to normal phase (IR to UV)
- Quench from normal to condensed phase (UV to IR)

Quantum quench and time dependence

Allow for time dependence of the Kondo coupling and study response of the condensate

Examples for time dependence of the Kondo coupling:

- Gaussian pulse in IR
- Quench from condensed to normal phase (IR to UV)
- Quench from normal to condensed phase (UV to IR)

Timescales governed by quasinormal modes (QNM)

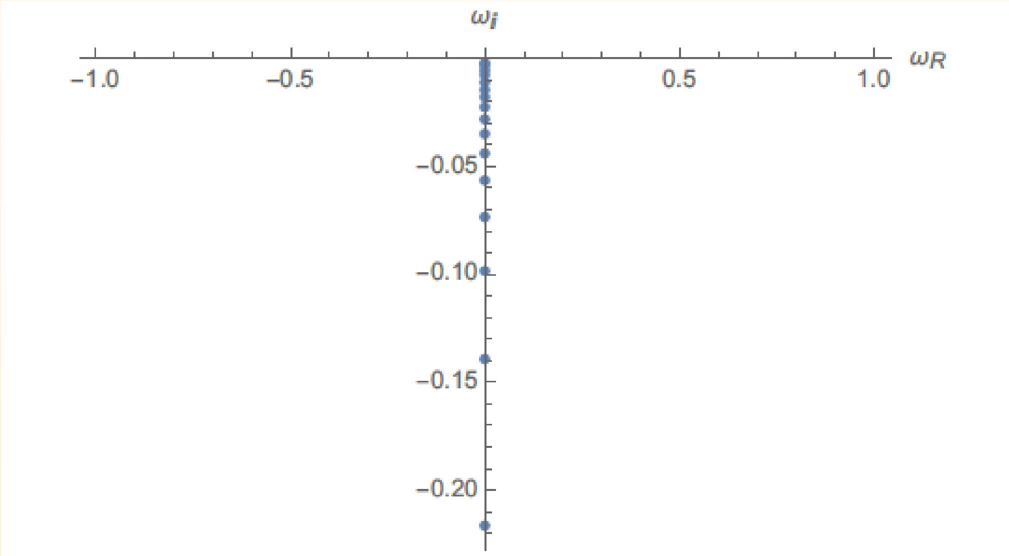
Complex eigenfrequencies of fluctuations in gravity system

Quasinormal modes

Complex eigenfrequencies ω_P of gravitational system determine time evolution

The ω_P also determine the poles in the Green's functions

In condensed phase:

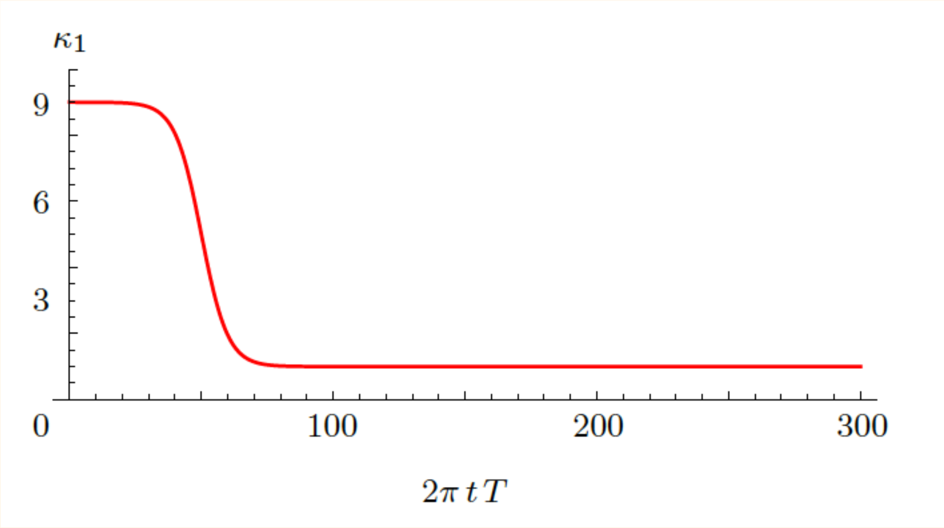


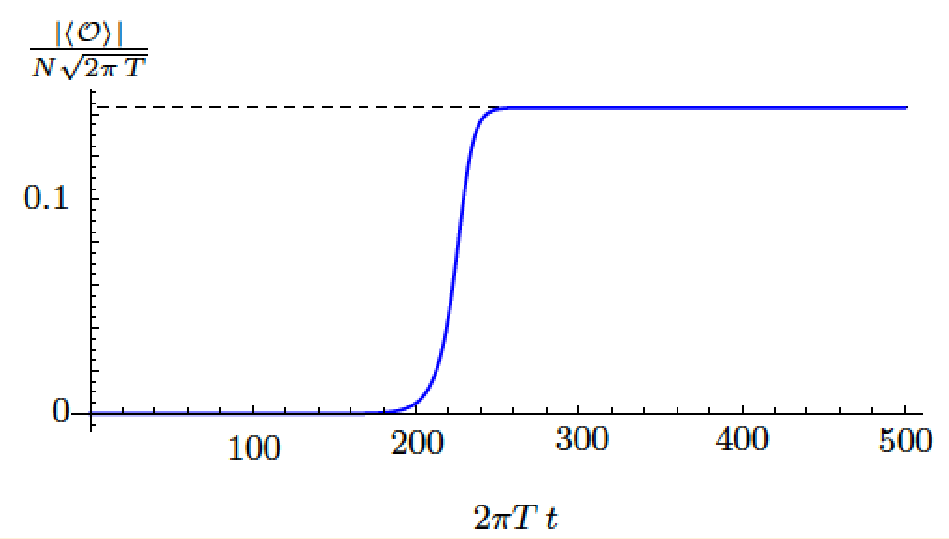
Quasinormal modes on negative imaginary axis, $\omega_{\rm pole} \propto -i\langle \mathcal{O} \rangle^2$

Kondo resonance

Quantum quench in Kondo model within gauge/gravity duality

J.E., Flory, Newrzella, Strydom, Wu JHEP (2017)





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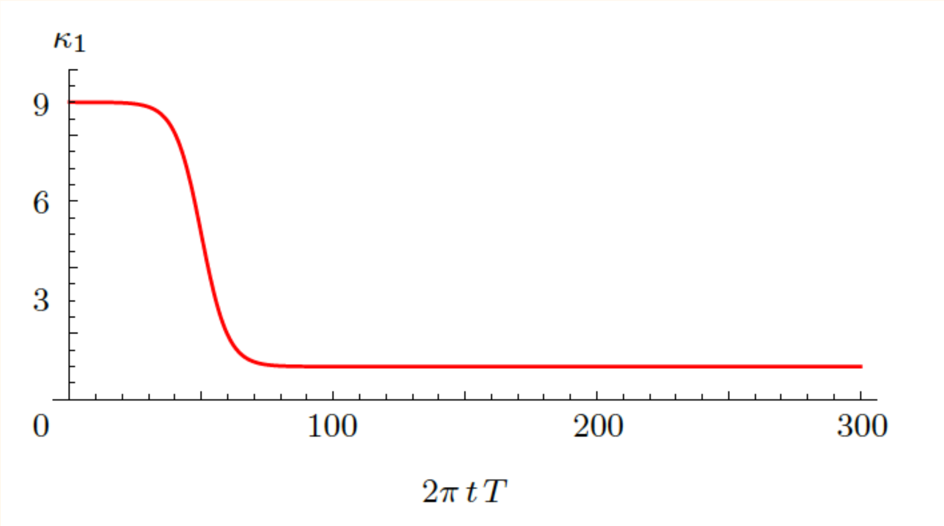
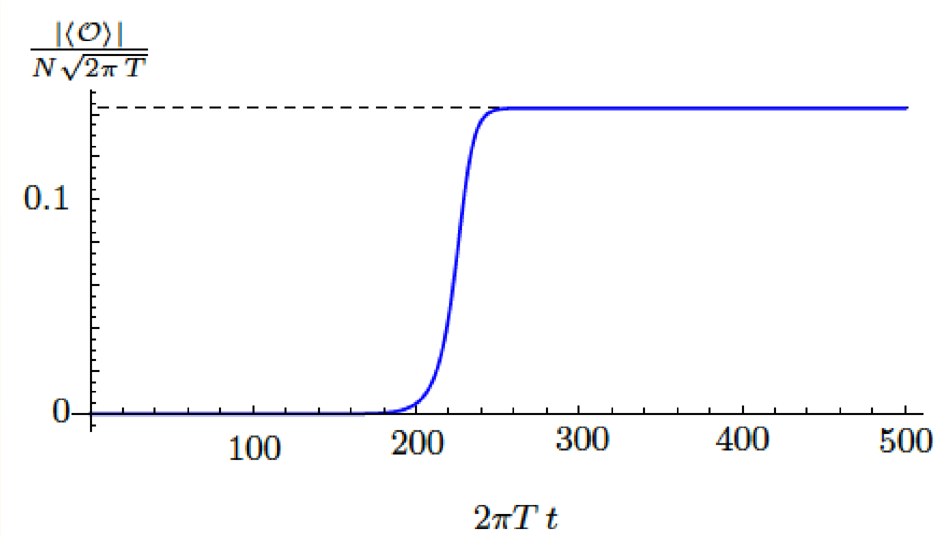
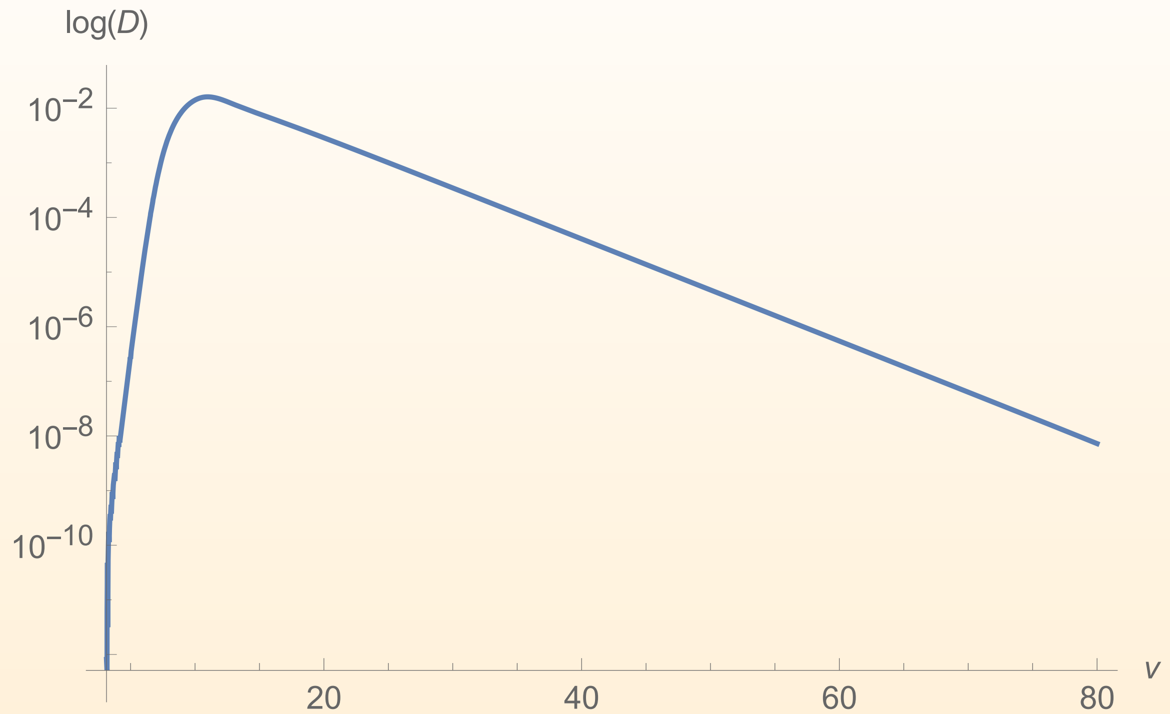




Formation of screening cloud:

Exponential fall-off of number of degrees of freedom at impurity

Screening happens exponentially fast



Flux at horizon (proportional to number of impurity degrees of freedom) as function of time

Relation to Sachdev-Ye-Kitaev model

Sachdev-Ye-Kitaev model: Gaussian random couplings $J_{\alpha\beta,\gamma\delta}$ Sachdev+Ye 1993, Kitaev 2015

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} J_{\alpha\beta,\gamma\delta} \, \chi_\alpha^\dagger \chi_\beta \chi_\gamma^\dagger \chi_\delta - \mu \sum_\alpha \chi_\alpha^\dagger \chi_\alpha$$

May be obtained from two-dimensional model as follows:

(Bray, Moore J. Phys. C 1980; Georges, Parcollet, Sachdev PRB 63 92001)

Reduction to single site by averaging over disorder

$$H_S = -\sum_{(ij)} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$S_{\text{eff}} = -\frac{J^2}{2N} \int_0^\beta d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau') \,, \qquad Q(\tau - \tau') = \frac{1}{N^2} \langle \vec{S}(\tau) \vec{S}(\tau') \rangle$$

Use Abrikosov fermions χ as before, $S^a = \chi^\dagger T^a \chi$, and take large N limit

Relation to Sachdev-Ye-Kitaev model

Similarly in Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192:

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$\Rightarrow$ Spectral asymmetry of Green's functions

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$\Rightarrow$ Spectral asymmetry of Green's functions

Sachdev 1506.05111, Phys. Rev. X 5, 041025 (2015):

Spectral asymmetry also observed in SYK model

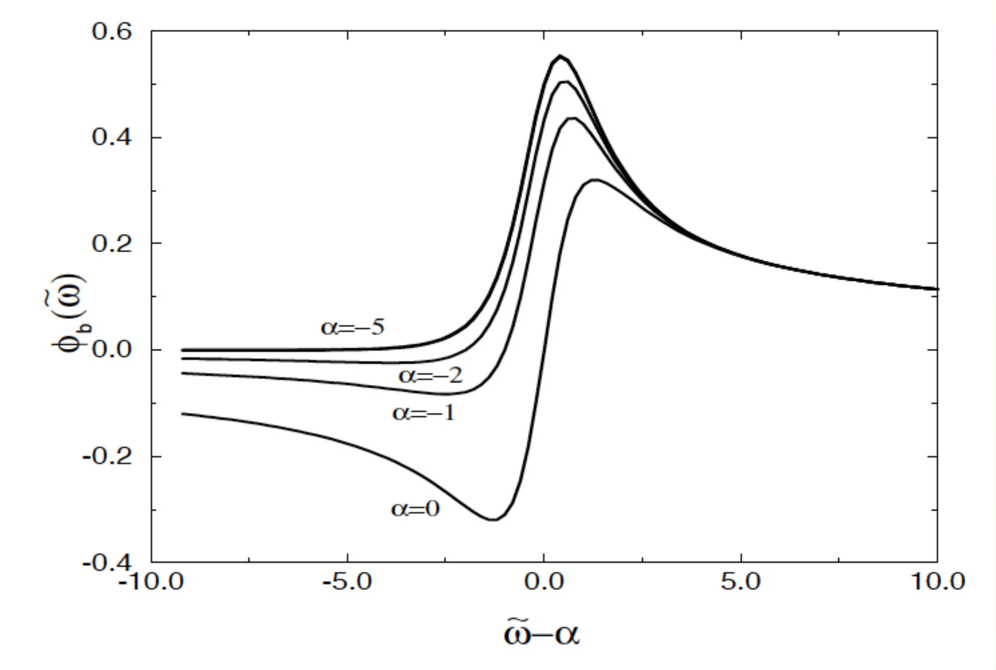
related to entropy of AdS_2 black hole

$$\omega_s = \frac{qT}{\hbar}\frac{\partial S}{\partial \mathcal{Q}}$$

Kondo model: Two-point functions at $T = 0$

Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192: Large N Kondo model

Spectral asymmetry ω_s: Particle-hole symmetry broken



$-\mathrm{Im}G^R$ for bosonic $\langle \mathcal{O}\mathcal{O}^\dagger \rangle$

Fano resonances

Example of Fano resonance

A related but different Fano resonance is observed in holographic model

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A related but different Fano resonance is observed in holographic model

Fano (1961):

A discrete set of resonant states interacts with a continuum of states

Example: Light scattering off an atom

Spectral function:

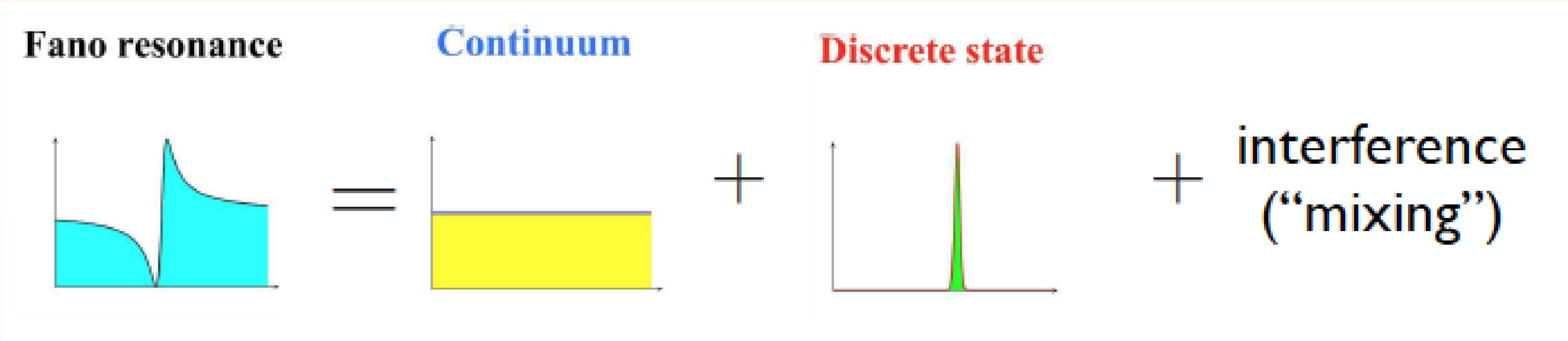
$$\rho_{\mathrm{Fano}}(\omega) = \frac{(\omega - \omega_0 + \frac{\Gamma}{2}q)^2}{(\omega - \omega_0)^2 + \left(\frac{\Gamma}{2}\right)^2}$$

q: Fano asymmetry parameter

$$q^2 \propto \frac{\text{Probability of resonant scattering}}{\text{Probability of non} - \text{resonant scattering}}$$

Fano resonances

$$\frac{(\omega-\omega_0+\frac{\Gamma}{2}q)^2}{(\omega-\omega_0)^2+\left(\frac{\Gamma}{2}\right)^2} = 1 + \frac{(q^2-1)\left(\frac{\Gamma}{2}\right)^2}{(\omega-\omega_0)^2+\left(\frac{\Gamma}{2}\right)^2} + \frac{2q\frac{\Gamma}{2}(\omega-\omega_0)}{(\omega-\omega_0)^2+\left(\frac{\Gamma}{2}\right)^2}$$



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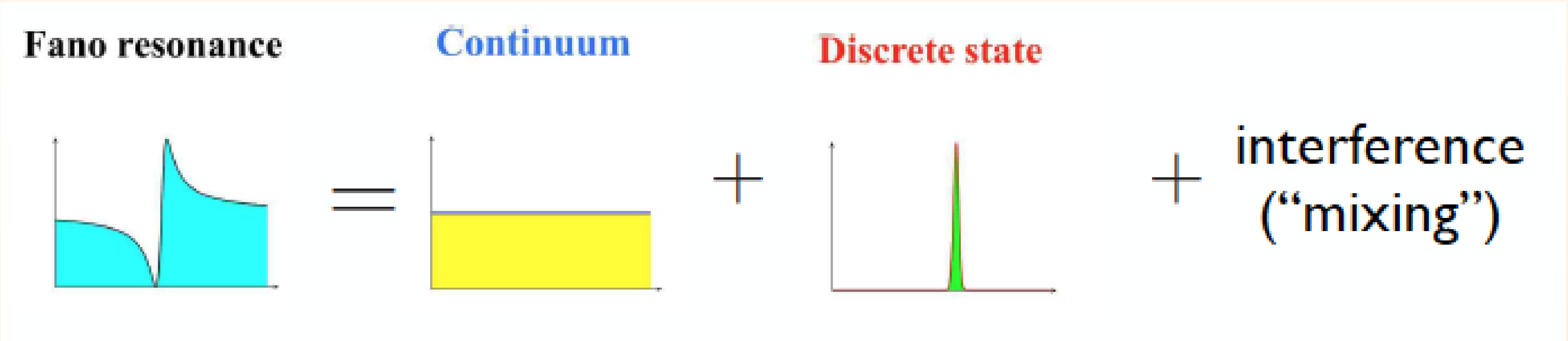
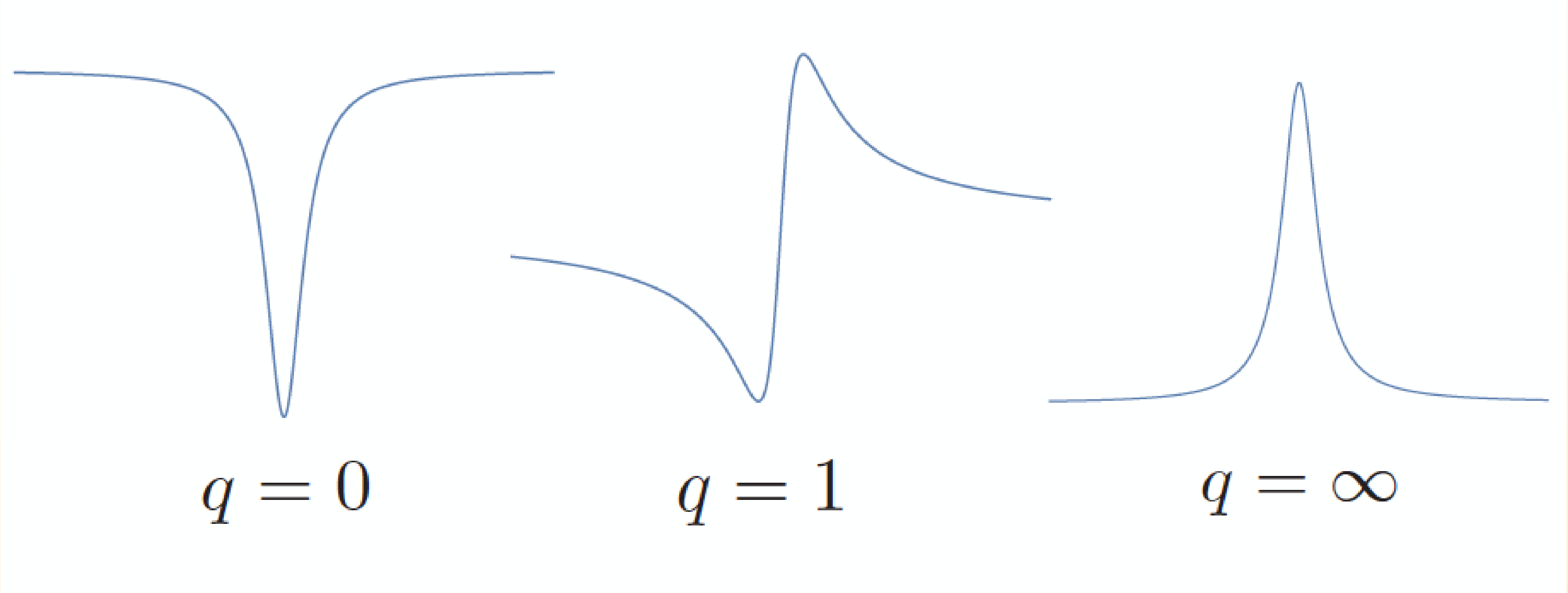


Observed in side-coupled quantum dots

Göres et al PRB 62 (2000) 2188

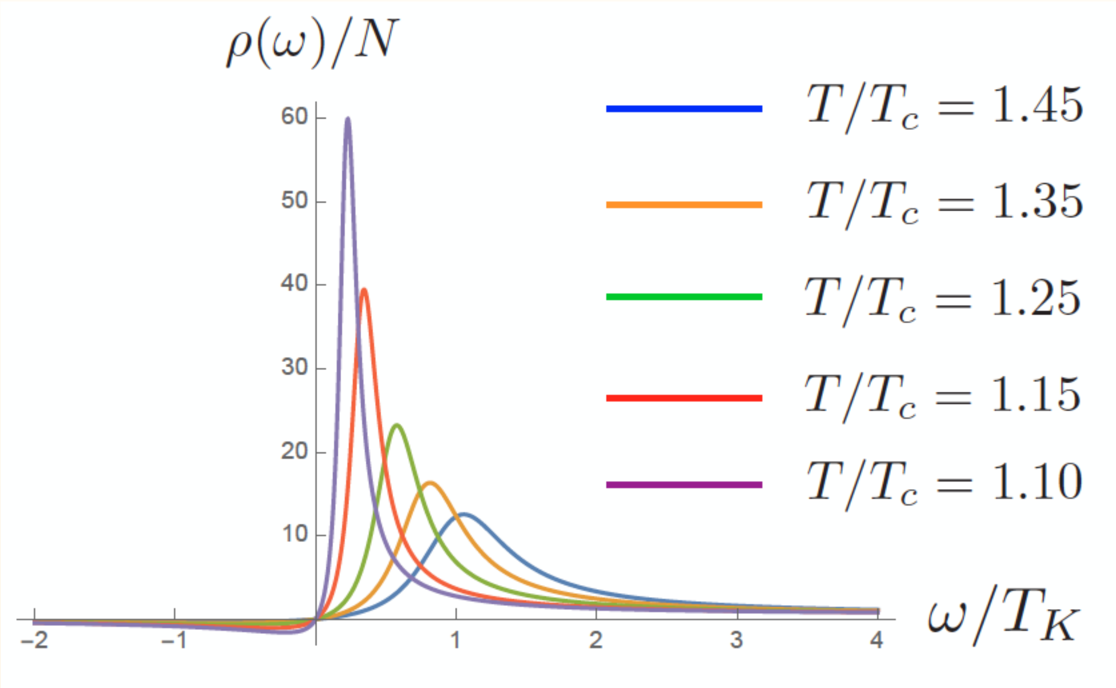
Fano resonance



$$q^2 \propto \frac{\text{Probability of resonant scattering}}{\text{Probability of non}-\text{resonant scattering}}$$

Spectral function $-\mathrm{Im}\langle\mathcal{O}^\dagger\mathcal{O}\rangle$ in normal phase, $\langle\mathcal{O}\rangle = 0$, $T > T_c$

J.E., Hoyos, O'Bannon, Papadimitriou, Probst, Wu JHEP 1703 (2017) 039 , PRD 96 (2017) 021901



$$\rho_{\mathrm{peak}} \propto \frac{1}{T - T_c}$$

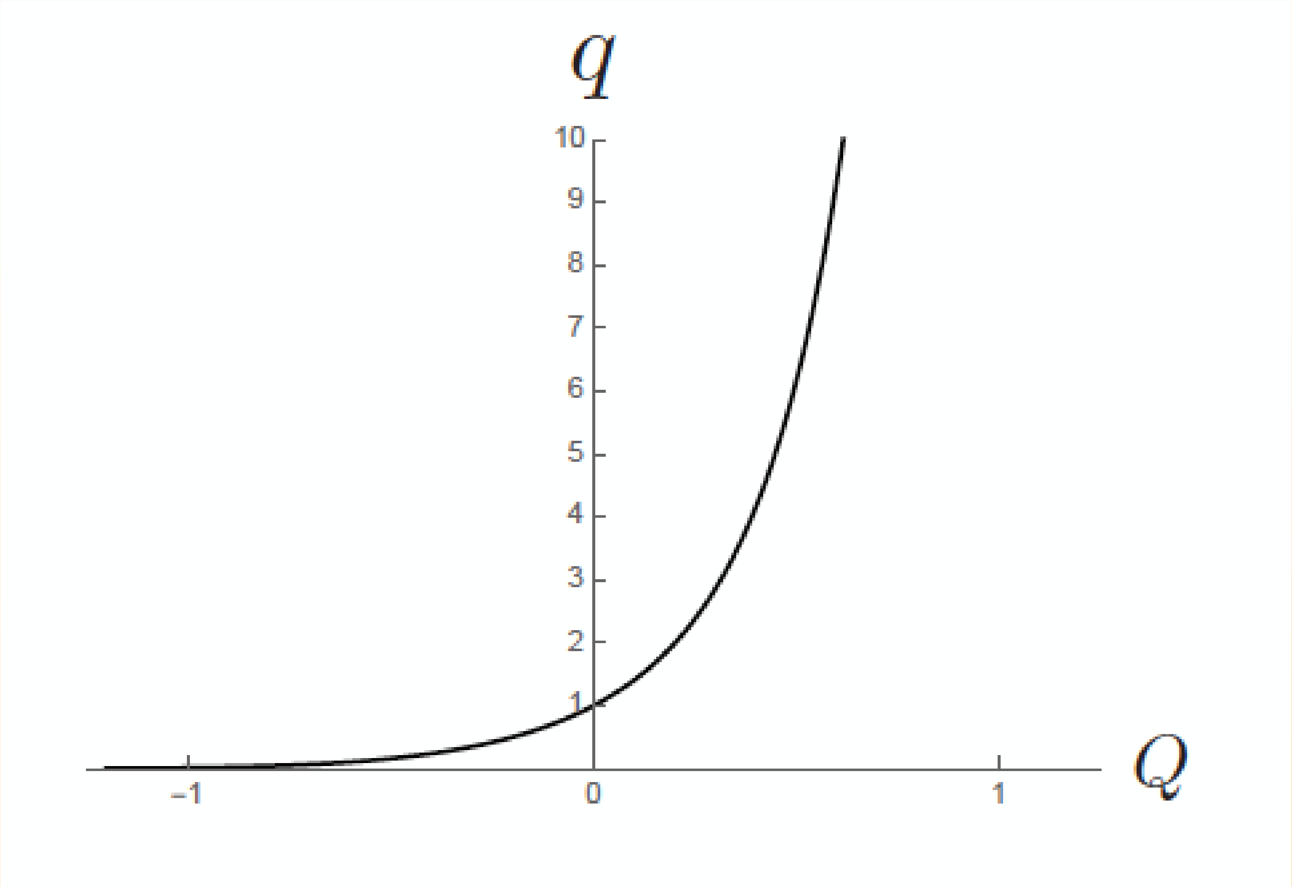
Fano resonance

Here: 0+1 CFT continuum $+$ Resonance with spin impurity $=$ Fano

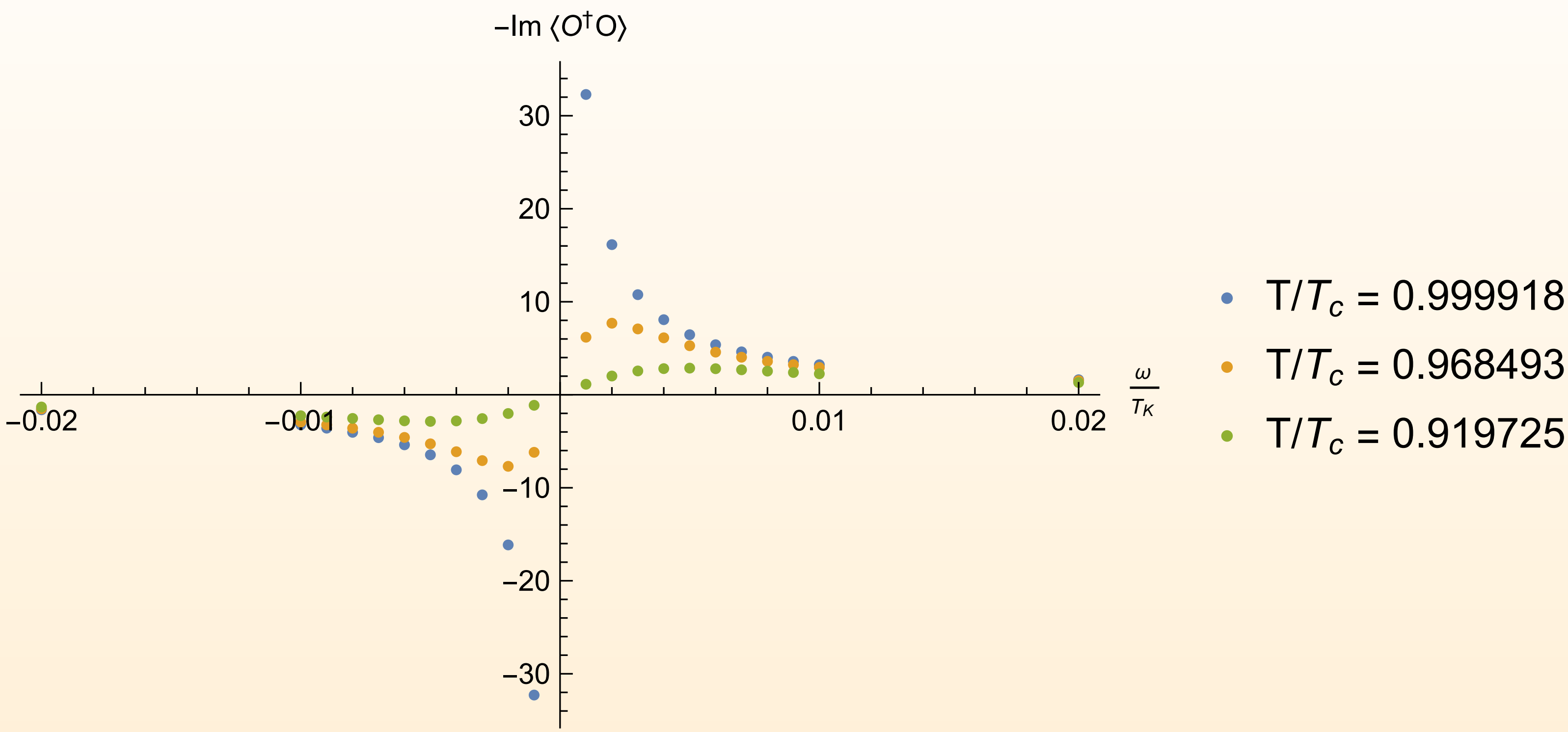
0+1-dimensional conformal symmetry of AdS_2 subspace
broken by double-trace operator of interaction with spin

Dependence of asymmetry parameter q on representation parameter Q

For $T \gtrsim T_c$:



Spectral function $-\mathrm{Im}\langle\mathcal{O}^\dagger\mathcal{O}\rangle$ in condensed phase, $\langle\mathcal{O}\rangle \neq 0$, $T < T_c$

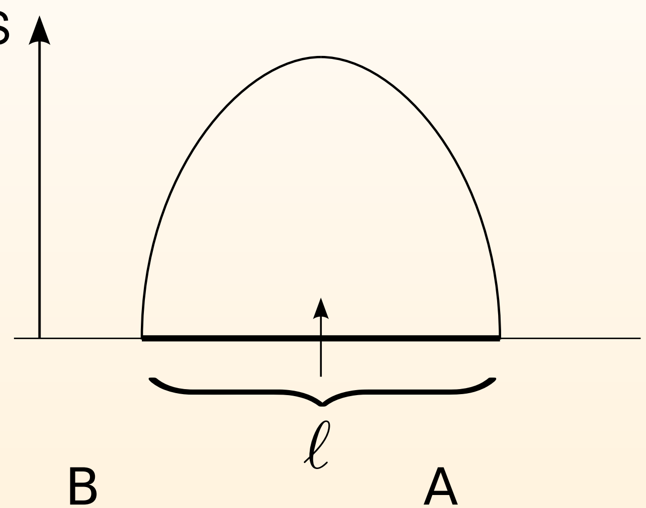


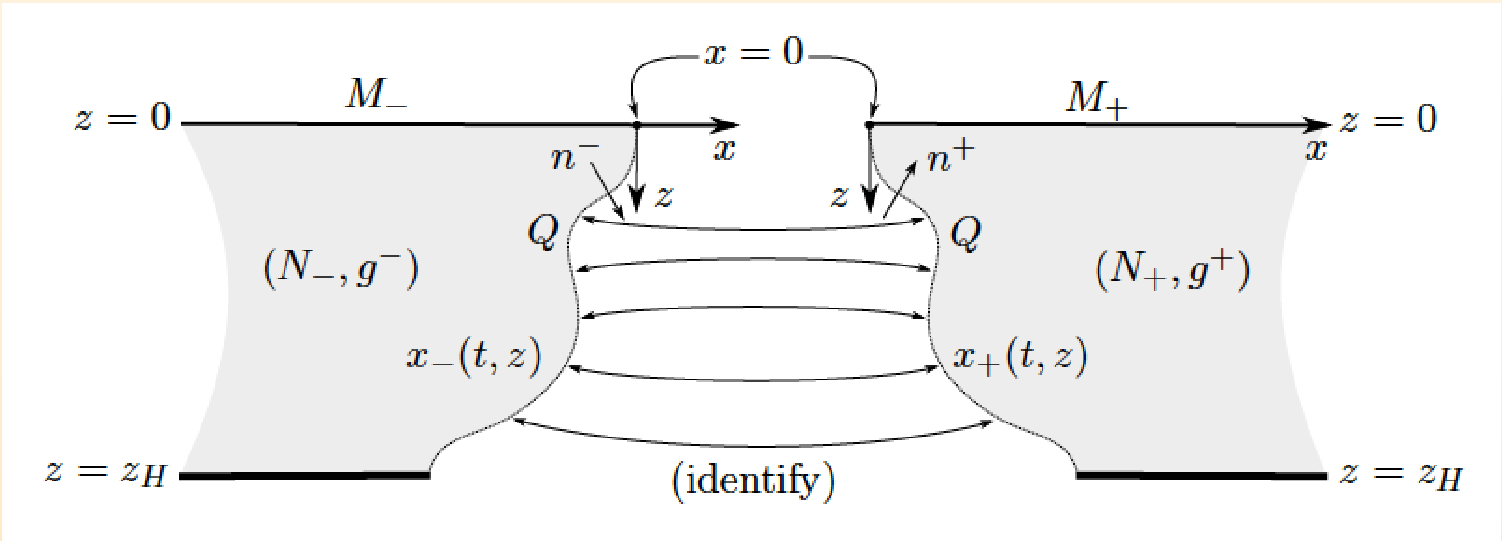
Fano asymmetry parameter $q = 1$ (i.e. no asymmetry)

Poles of retarded Green's function purely imaginary, $\omega \propto -i|\langle\mathcal{O}\rangle|^2$

Manifestation of large N Kondo resonance

$$S_{\text{imp}} = S_{\text{impurity present}} - S_{\text{impurity absent}}$$





Entanglement entropy for magnetic impurity: Comparison to field theory

Field theory result: Sorensen, Chang, Laflorencie, Affleck 2007 , (Eriksson, Johannesson 2011)

$$\Delta S_{\text{imp}}(\ell) = \frac{\pi^2 \xi_K T}{6} \coth(2\pi \ell T) + C_0$$

Entanglement entropy for magnetic impurity: Comparison to field theory

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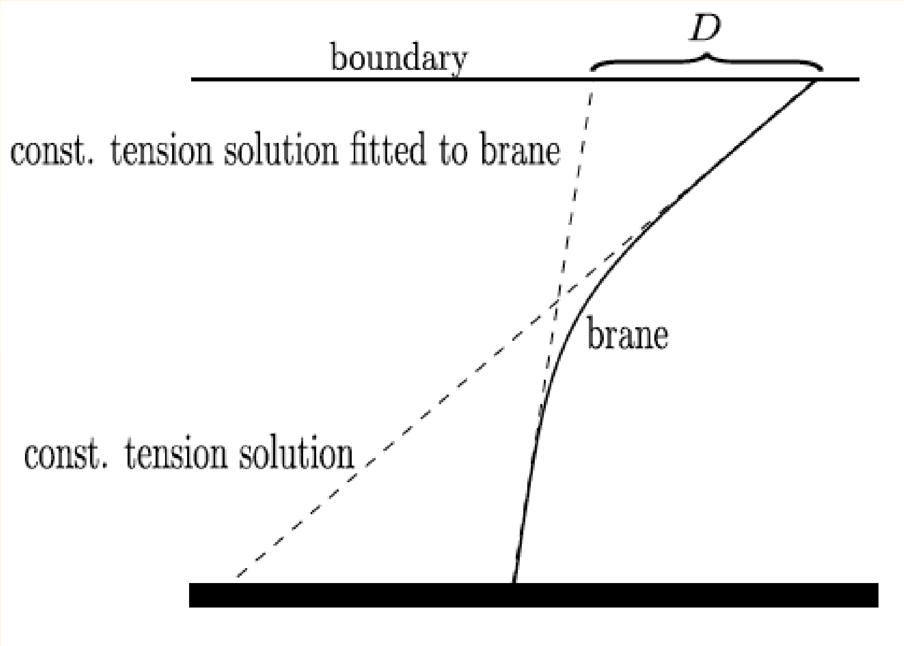
$$\Delta S_{\text{imp}}(\ell) = \frac{\pi^2 \xi_K T}{6} \coth(2\pi \ell T) + C_0$$

In our gravity approach: Same result if $D \propto \xi_k$



Conclusions and outlook

- Kondo model:
- Magnetic impurity coupled to strongly coupled system
- Quantum quenches
  - Dominated by quasinormal modes
- Two-point functions
  - Spectral asymmetry
  - Relation to SYK model
- Entanglement entropy
  - In agreement with g-theorem
  - Reproduces large N field theory result for large ℓ
  - Geometrical realization of Kondo correlation length
- The model suggests new models within string theory
  Gravity dual of Kondo model mapping 2d CFT's to string theory in $AdS_3 \times X$

Würzburg: Conference ‘Gauge/Gravity Duality 2018’: 30th July - 3rd August

https://www.physik.uni-wuerzburg.de/en/tp3/gaugegravity_duality_2018/