Quantum quenches and Fano resonances
in holographic strange metals with magnetic impurities

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Kondo models from gauge/gravity duality
Kondo effect:

Screening of a magnetic impurity by conduction electrons at low temperatures
Kondo models from gauge/gravity duality

Kondo effect:

**Screening of a magnetic impurity by conduction electrons at low temperatures**

Motivation for study within gauge/gravity duality:
Kondo models from gauge/gravity duality

Kondo effect:

Screening of a magnetic impurity by conduction electrons at low temperatures

Motivation for study within gauge/gravity duality:

1. New applications of gauge/gravity duality to condensed matter physics:
   - Magnetic impurity coupled to strongly correlated electron system
   - Entanglement entropy, quantum quenches

2. Simple model for a RG flow with dynamical scale generation (as in QCD)

3. Example for holographic $g$-theorem

4. Relation to Sachdev-Ye-Kitaev model
Kondo models from holography

- **Model**
  J.E., Hoyos, O’Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

- **Entanglement entropy**
  J.E., Flory, Hoyos, Newrzella, O’Bannon, Wu 1511.03666, Fortsch.Phys. 64 (2016)

- **Two-point functions**

- **Quantum quenches**
  J.E., Flory, Newrzella, Wu JHEP 1704 (2017) 045
Kondo effect

Free electrons + impurity spin

Impurity screened
Kondo model

Original Kondo model (Kondo 1964):
Magnetic impurity interacting with free electron gas

Hamiltonian:

\[ H = \frac{v_F}{2\pi} \psi^\dagger i\partial_x \psi + \lambda_K v_F \delta(x) \vec{S} \cdot \vec{J}, \quad \vec{J} = \psi^\dagger \frac{1}{2} \vec{T} \psi \]

Decisive in development of renormalization group
IR fixed point, CFT approach Affleck, Ludwig ’90’s

Solution using Bethe ansatz Andrei, Wiegmann 80’s

Large \( N \) Kondo model Read, Newns, Coleman, ... 80’s
Scattering with magnetic impurities

\[ \rho \sim \rho_0 \left( 1 + \kappa \log \frac{T}{|\epsilon - \epsilon_F|} \right) \]

Antiferromagnetic coupling \( \kappa < 0 \)
Fig. 1. Comparison of experimental and theoretical $\rho$-$T$ curves for dilute AuFe alloys.
Breakdown of perturbation theory

\[ \rho \sim \rho_0 \left( 1 + \kappa \log \frac{T}{|\epsilon - \epsilon_F|} \right) \]

Perturbation theory breaks down at \( T_K = |\epsilon - \epsilon_F|e^{1/\kappa} \)

\( T_K \): Kondo temperature
Breakdown of perturbation theory

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\( T_K \): Kondo temperature

\( T_K \sim \Lambda_{\text{QCD}} \)
Kondo models from gauge/gravity duality

Gauge/gravity requires large $N$: Spin group $SU(N)$
Kondo models from gauge/gravity duality

Gauge/gravity requires large $N$: Spin group $SU(N)$

In this case, interaction term simplifies introducing slave fermions:

$$S^a = \chi^\dagger T^a \chi$$

Totally antisymmetric representation: Young tableau with $Q$ boxes

Constraint: $\chi^\dagger \chi = Q$

Interaction: $J^a S^a = (\psi^\dagger T^a \psi) (\chi^\dagger T^a \chi) = \mathcal{O} \mathcal{O}^\dagger$, where $\mathcal{O} = \psi^\dagger \chi$

Screened phase has condensate $\langle \mathcal{O} \rangle$

Coleman PRB 35, 5072 (1987)
Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192, PRB 58 (1998) 3794
Senthil, Sachdev, Vojta cond-mat/0209144, PRL 90 (2003) 216403
Kondo models from gauge/gravity duality

J.E., Hoyos, O’Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Coupling of a magnetic impurity to a strongly interacting non-Fermi liquid
Coupling of a magnetic impurity to a strongly interacting non-Fermi liquid

Results:

- RG flow from perturbation by ‘double-trace’ operator
- Dynamical scale generation, screening
- Holographic superconductor: Condensate forms in $AdS_2$
- Power-law scaling of resistivity in IR with real exponent
- Holographic entanglement entropy from including backreaction
- Quantum quench: Equilibration dominated by quasinormal modes
- Fano resonance in spectral function (spectral asymmetry)
Kondo models from gauge/gravity duality

J.E., Hoyos, O’Bannon, Wu 1310.3271, JHEP 1312 (2013) 086
Top-down brane realization

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- 3-7 strings: Chiral fermions $\psi$ in 1+1 dimensions
- 3-5 strings: Slave fermions $\chi$ in 0+1 dimensions
- 5-7 strings: Scalar (tachyon)
Near-horizon limit and field-operator map

D3: $AdS_5 \times S^5$

D7: $AdS_3 \times S^5 \rightarrow$ Chern-Simons $A_\mu$ dual to $J^\mu = \psi^\dagger \sigma^\mu \psi$

D5: $AdS_2 \times S^4 \rightarrow \begin{cases} \text{YM } a_t \text{ dual to } \chi^\dagger \chi = q \\ \text{Scalar dual to } \psi^\dagger \chi \end{cases}$

<table>
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<tr>
<th>Operator</th>
<th>Gravity field</th>
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<tr>
<td>Electron current $J$</td>
<td>$\Leftrightarrow$ Chern-Simons gauge field $A$ in $AdS_3$</td>
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<tr>
<td>Charge $Q = \chi^\dagger \chi$</td>
<td>$\Leftrightarrow$ 2d gauge field $a$ in $AdS_2$</td>
</tr>
<tr>
<td>Operator $O = \psi^\dagger \chi$</td>
<td>$\Leftrightarrow$ 2d complex scalar $\Phi$</td>
</tr>
</tbody>
</table>
Bottom-up gravity dual for Kondo model

Action:

\[ S = S_{\text{Einstein-Hilbert}} + S_{CS} + S_{AdS_2}, \]
\[ S_{CS} = -\frac{N}{4\pi} \int_{AdS_3} \text{Tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right), \]
\[ S_{AdS_2} = -N \int dx dt dz \delta(x) \sqrt{-g} \left[ \frac{1}{4} \text{Tr} f_{mn} f_{mn} + g^{mn} (D_m \Phi)^\dagger D_n \Phi + V(\Phi^\dagger \Phi) \right] \]
\[ V(\Phi) = M^2 \Phi^\dagger \Phi \]

Metric: BTZ black hole

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{1}{z^2} \left( \frac{dz^2}{h(z)} - h(z) dt^2 + dx^2 \right), \]
\[ h(z) = 1 - z^2/z_H^2, \quad T = 1/(2\pi z_H) \]

AdS_2 gauge field: asymptotically \[ a_t = \frac{Q/N}{z} + \mu \]
Boundary expansion

\[ \Phi = z^{1/2}(\alpha \ln z + \beta) \]

\[ \alpha = \kappa \beta \]

\( \kappa \) dual to double-trace deformation

Witten hep-th/0112258

Berkooz, Sever, Shomer hep-th/0112264
Boundary expansion

\[ \Phi = z^{1/2}(\alpha \ln z + \beta) \]

\[ \alpha = \kappa \beta \]

\( \kappa \) dual to double-trace deformation

\( \Phi \) invariant under renormalization \( \Rightarrow \) Running coupling

\[ \kappa_T = \frac{\kappa_0}{1 + \kappa_0 \ln \left( \frac{\Lambda}{2\pi T} \right)} \]

Dynamical scale generation
Divergence of Kondo coupling determines Kondo temperature $T_K$

Transition temperature to phase with condensed scalar: $T_c$

$T_c < T_K$
Kondo models from gauge/gravity duality

RG flow

UV
Strongly interacting electrons

Deformation by Kondo operator

IR
Strongly interacting electrons

Non-trivial condensate
Kondo models from gauge/gravity duality

Normalized condensate $\langle \mathcal{O} \rangle \equiv \kappa \beta$ as function of the temperature

(a) Mean field transition

$\langle \mathcal{O} \rangle$ approaches constant for $T \to 0$
\[ \sqrt{-g} f^{tr} \bigg|_{\partial \text{AdS}_2} = q \]

charge \( q = Q/N \) of 2d gauge field determines impurity representation

Impurity is screened
Allow for time dependence of the Kondo coupling and study response of the condensate
Quantum quench and time dependence

Allow for time dependence of the Kondo coupling and study response of the condensate

Examples for time dependence of the Kondo coupling:

- Gaussian pulse in IR
- Quench from condensed to normal phase (IR to UV)
- Quench from normal to condensed phase (UV to IR)
Quantum quench and time dependence

Allow for time dependence of the Kondo coupling and study response of the condensate

Examples for time dependence of the Kondo coupling:

- Gaussian pulse in IR
- Quench from condensed to normal phase (IR to UV)
- Quench from normal to condensed phase (UV to IR)

Timescales governed by quasinormal modes (QNM)

Complex eigenfrequencies of fluctuations in gravity system
Quasinormal modes

Complex eigenfrequencies $\omega_P$ of gravitational system determine time evolution

The $\omega_P$ also determine the poles in the Green’s functions

In condensed phase:

Quasinormal modes on negative imaginary axis, $\omega_{\text{pole}} \propto -i \langle \mathcal{O} \rangle^2$

Kondo resonance
Quantum quench in Kondo model within gauge/gravity duality

J.E., Flory, Newrzella, Strydom, Wu JHEP (2017)
Formation of screening cloud:

Exponential fall-off of number of degrees of freedom at impurity

Quantum quench in Kondo model within gauge/gravity duality

J.E., Flory, Newrzella, Strydom, Wu JHEP (2017)
Screening happens exponentially fast

Flux at horizon (proportional to number of impurity degrees of freedom) as function of time
Sachdev-Ye-Kitaev model: Gaussian random couplings $J_{\alpha\beta,\gamma\delta}$  

$$ H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} J_{\alpha\beta,\gamma\delta} \chi_{\alpha}^{\dagger} \chi_{\beta} \chi_{\gamma} \chi_{\delta} - \mu \sum_{\alpha} \chi_{\alpha}^{\dagger} \chi_{\alpha} $$

May be obtained from two-dimensional model as follows:

(Bray, Moore J. Phys. C 1980; Georges, Parcollet, Sachdev PRB 63 92001)

Reduction to single site by averaging over disorder

$$ H_{S} = - \sum_{(ij)} J_{ij} \vec{S}_{i} \cdot \vec{S}_{j} $$

$$ S_{\text{eff}} = - \frac{J^{2}}{2N} \int_{0}^{\beta} d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau'), \quad Q(\tau - \tau') = \frac{1}{N^{2}} \langle \vec{S}(\tau) \vec{S}(\tau') \rangle $$

Use Abrikosov fermions $\chi$ as before, $S^{a} = \chi^{\dagger} T^{a} \chi$, and take large $N$ limit
Similarly in Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192:

Reduction of large $N$-Kondo model to single-site model by integrating out conduction electrons
Relation to Sachdev-Ye-Kitaev model

Similarly in Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192:

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⇒ Spectral asymmetry of Green’s functions
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Reduction of large $N$-Kondo model to single-site model by integrating out conduction electrons

⇒ Spectral asymmetry of Green’s functions


Spectral asymmetry also observed in SYK model related to entropy of AdS$_2$ black hole

$$\omega_s = \frac{qT}{\hbar} \frac{\partial S}{\partial Q}$$
Kondo model: Two-point functions at $T = 0$

Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192: Large $N$ Kondo model

Spectral asymmetry $\omega_s$: Particle-hole symmetry broken

$-\text{Im} G^R$ for bosonic $\langle \mathcal{O} \mathcal{O}^\dagger \rangle$
Fano resonances

Example of Fano resonance

A related but different Fano resonance is observed in holographic model
Example of Fano resonance

A related but different Fano resonance is observed in holographic model

Fano (1961):
A discrete set of resonant states interacts with a continuum of states

Example: Light scattering off an atom

Spectral function:

$$\rho_{\text{Fano}}(\omega) = \frac{(\omega - \omega_0 + \frac{\Gamma}{2} q)^2}{(\omega - \omega_0)^2 + \left(\frac{\Gamma}{2}\right)^2}$$

$q$: Fano asymmetry parameter

$$q^2 \propto \frac{\text{Probability of resonant scattering}}{\text{Probability of non–resonant scattering}}$$
Fano resonances

\[
\frac{\left(\omega - \omega_0 + \frac{\Gamma}{2}q\right)^2}{\left(\omega - \omega_0\right)^2 + \left(\frac{\Gamma}{2}\right)^2} = 1 + \frac{(q^2 - 1)\left(\frac{\Gamma}{2}\right)^2}{\left(\omega - \omega_0\right)^2 + \left(\frac{\Gamma}{2}\right)^2} + \frac{2q\frac{\Gamma}{2}(\omega - \omega_0)}{\left(\omega - \omega_0\right)^2 + \left(\frac{\Gamma}{2}\right)^2}
\]

Fano resonance

Continuum

Discrete state

interference (“mixing”)

Fano resonances

\[
\frac{(\omega - \omega_0 + \frac{\Gamma}{2}q)^2}{(\omega - \omega_0)^2 + \left(\frac{\Gamma}{2}\right)^2} = 1 + \frac{(q^2 - 1)\left(\frac{\Gamma}{2}\right)^2}{(\omega - \omega_0)^2 + \left(\frac{\Gamma}{2}\right)^2} + \frac{2q\frac{\Gamma}{2}(\omega - \omega_0)}{(\omega - \omega_0)^2 + \left(\frac{\Gamma}{2}\right)^2}
\]

Observed in side-coupled quantum dots

Göres et al PRB 62 (2000) 2188
Fano resonance

\[ q^2 \propto \frac{\text{Probability of resonant scattering}}{\text{Probability of non-resonant scattering}} \]
Spectral function $-\text{Im}\langle \mathcal{O}^\dagger \mathcal{O} \rangle$ in normal phase, $\langle \mathcal{O} \rangle = 0$, $T > T_c$

\[ \rho_{\text{peak}} \propto \frac{1}{T - T_c} \]

Fano resonance

Here: 0+1 CFT continuum + Resonance with spin impurity = Fano

0+1-dimensional conformal symmetry of AdS$_2$ subspace
broken by double-trace operator of interaction with spin
For $T \gtrsim T_c$:
Spectral function $-\text{Im}\langle \mathcal{O}^\dagger \mathcal{O} \rangle$ in condensed phase, $\langle \mathcal{O} \rangle \neq 0$, $T < T_c$

Fano asymmetry parameter $q = 1$ (i.e. no asymmetry)

Poles of retarded Green’s function purely imaginary, $\omega \propto -i|\langle \mathcal{O} \rangle|^2$

Manifestation of large $N$ Kondo resonance
Impurity entropy

\[ S_{\text{imp}} = S_{\text{impurity present}} - S_{\text{impurity absent}} \]
Field theory result: \( \Delta S_{\text{imp}}(\ell) = \frac{\pi^2 \xi_K T}{6} \coth(2\pi \ell T) + C_0 \)
Entanglement entropy for magnetic impurity: Comparison to field theory

Field theory result: Sorensen, Chang, Laflorence, Affleck 2007, (Eriksson, Johannesson 2011)

\[ \Delta S_{\text{imp}}(\ell) = \frac{\pi^2 \xi_K T}{6} \coth(2\pi \ell T) + C_0 \]

In our gravity approach: Same result if \( D \propto \xi_k \)
Conclusions and outlook

- **Kondo model:**
  - Magnetic impurity coupled to strongly coupled system
  - Quantum quenches
    - Dominated by quasinormal modes
- **Two-point functions**
  - Spectral asymmetry
  - Relation to SYK model
- **Entanglement entropy**
  - In agreement with $g$-theorem
  - Reproduces large $N$ field theory result for large $\ell$
  - Geometrical realization of Kondo correlation length
- The model suggests new models within string theory
  - Gravity dual of Kondo model mapping 2d CFT’s to string theory in $\text{AdS}_3 \times X$