Introduction to hydrodynamics and electronic fluids

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Goal of the lecture and Acknowledgments

- The goal of this lecture is to give a short introduction to non-quasiparticle approaches to transport at strong coupling, e.g. hydrodynamics, memory matrices and AdS/CFT.
- After setting up the stage, I will mostly focus on momentum relaxation in metallic phases.
- I will also mention the possibility of fundamental bounds on transport coefficients.

Main references

- *Lectures on hydrodynamics*, Pavel Kovtun, [arXiv:1205.5040].
- *Holographic quantum matter*, Sean Hartnoll, Andrew Lucas and Subir Sachdev, [arXiv:1612.07324].

Transport with long-lived quasiparticles

- Transport in a weakly-coupled metallic phase is accounted for by tracking the dynamics of the weakly-interacting quasiparticles.

- Infinite number of quasi-conserved quantities $\tau_{qp} \gg \hbar/(k_B T)$ (or $\tau_{el} \ll \tau_{inel}$).

- Kinetic Boltzmann equation: captures the dynamics of $n_{\delta k}$, the qp density at wavector $\delta k = k - k_F$. Difficulty: solving the collision integral but this is a technical obstacle, not a conceptual one.

- From the point of view of transport, this typically means that the ac conductivity

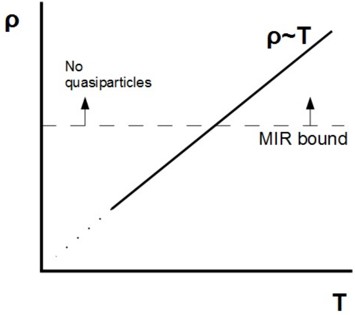
  $$\sigma(\omega, k=0) \sim \frac{\omega_p^2}{\Gamma - i\omega}, \quad \Gamma = \frac{1}{\tau_{qp}}$$

  There is a sharp Drude-like peak at $\omega = 0$ and

  $$\sigma_{dc} = \lim_{\omega \to 0} \sigma(\omega) = \frac{ne^2\tau_{qp}}{m} \gg \frac{1}{T}$$

  This can be taken as an operational definition of a good metal.

The MIR bound



- The qp mean free path is bounded from below by Quantum Mechanics:

$$k_F \ell \gtrsim \hbar$$

- This implies a lower bound on the conductivity of a good metal

$$\sigma_{dc} = \frac{ne^2 \tau_{qp}}{m} \gtrsim \frac{e^2}{\hbar}$$

- This can also be reformulated using the uncertainty principle on energy

$$E_F k_B T \gtrsim \hbar \quad \Rightarrow \quad \sigma_{dc} \gtrsim \frac{E_F}{k_B T} \frac{e^2}{\hbar}$$

Transport without long-lived quasiparticles

- What about cases without long-lived quasiparticles $\tau_{qp} \sim 1/T$?

- Specifically, I will focus here on cases with an emerging long lived collective mode: momentum (tomorrow, Goldstone boson as well).

- Hydrodynamics: relaxation towards equilibrium $\tau \gg \tau_{th} \sim 1/T$. Expansion in small gradients which encapsulates the assumption that $\tau/\tau_{th} \gg 1$ or equivalently $\xi/\ell_{mfp} \gg 1$.

- The memory matrix formalism does not assume small gradients: 'disorder' can vary importantly on microscopic scales. However it is only practically useful if there is only a small number of long-lived operators.

- AdS/CFT gives results consistent with both previous approaches, and allows to describe the crossover from weak to strong breaking.

Hydrodynamics of clean electronic fluids

- The starting point is conservation equations for energy (entropy), momentum and charge densities (symmetries).

$$\partial_t s + \partial_i \left(\frac{j_q^i}{T} \right) = 0\,, \quad \partial_t \pi^i + \partial_j \tau^{ij} = 0\,, \quad \partial_t \rho + \partial_i j^i = 0$$

- Next, we add an applied electric field $E_i \sim O(\partial)$:

$$\partial_t \delta s + \partial_i \left(\frac{j_q^i}{T} \right) = \frac{E_i j^i}{T}\,, \quad \partial_t \pi^i + \partial_j \tau^{ij} = \rho \left(E^i + v_k F^{ki} \right)$$

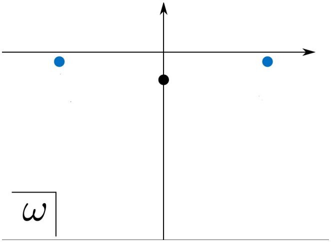
- We give a constitutive relation to currents order by order in gradients

$$j^i = \rho v^i - \sigma_o \left(\partial^i \mu - E^i \right) - \alpha_o \partial_i T + O(\partial^2)\,,$$

$$j_q^i = sTv^i - T\alpha_o \left(\partial^i \mu - E^i \right) - T\bar{\kappa}_o \partial_i T + O(\partial^2)\,,$$

$$\tau^{ij} = p\delta^{ij} - \eta \left(\partial^i v^j + \partial^j v^i \right) + (\zeta - \eta)\, \partial_k v^k \delta^{ij} + O(\partial^2)$$

Spectrum of modes



- There are three longitudinal modes: two acoustic and a diffusive mode

$$\omega_\pm = \pm c_s k - i\gamma_s k^2 \,, \quad \omega_{inc} = -iD_{inc} k^2$$

- The sound modes are carried by momentum and pressure fluctuations

$$G^R_{\pi\pi}, G^R_{\delta p \delta p} \sim \frac{1}{\omega^2 - c_s^2 k^2 - 2i\gamma_s k^2}$$

- The diffusive mode is carried by a combination of charge and entropy

$$G^R_{\delta\rho_{inc}\delta\rho_{inc}} \sim \frac{1}{\omega + iD_{inc} k^2} \,, \quad \delta\rho_{inc} = s\delta\rho - \rho\delta s$$

Thermoelectric conductivities

- Finally, solve for linearized fluctuations in terms of E_i, using the relations between vevs and sources

$$\pi^i = \chi_{PP} v^i ,$$

$$\begin{pmatrix} \delta\rho \\ \delta s \end{pmatrix} = \begin{pmatrix} \chi_{\rho\rho} & \chi_{\rho s} \\ \chi_{\rho s} & \chi_{ss} \end{pmatrix} \begin{pmatrix} \delta\mu \\ \delta T \end{pmatrix}$$

- Generalized Ohm's law

$$\begin{pmatrix} j \\ j_q \end{pmatrix} = \begin{pmatrix} \sigma & T\alpha \\ T\alpha & T\bar{\kappa} \end{pmatrix} \begin{pmatrix} E \\ -\partial\delta T/T \end{pmatrix}$$

Thermoelectric conductivities

$$\sigma(\omega) = \sigma_o + \frac{\rho^2}{\chi_{PP}} \left(\frac{i}{\omega} + \pi\delta(\omega) \right)$$

$$\alpha(\omega) = \alpha_o + \frac{\rho s}{\chi_{PP}} \left(\frac{i}{\omega} + \pi\delta(\omega) \right)$$

$$\bar{\kappa}(\omega) = \bar{\kappa}_o + \frac{s^2 T}{\chi_{PP}} \left(\frac{i}{\omega} + \pi\delta(\omega) \right)$$

- Their dc limit $\omega \to 0$ is formally infinite. This is due to momentum conservation and the non-zero overlap between the electric and heat currents with momentum:

$$\chi_{JP} = \frac{\delta j^i}{\delta v^i} = \rho$$

$$\chi_{J_Q P} = \frac{\delta j_q^i}{\delta v^i} = sT$$

Finite dc thermal conductivity

- Consider the heat conductivity with open circuit boundary conditions

$$\kappa \equiv T \frac{\delta j_q}{\delta \partial T}\bigg|_{j=0} = \bar{\kappa} - \frac{\alpha^2}{T\sigma}$$

- Finite as $\omega \to 0$

$$\kappa = \bar{\kappa}_o - \frac{2sT}{\rho}\alpha_o + \frac{s^2 T}{\rho^2}\sigma_o$$

- The open circuit boundary conditions remove the contribution of the sound modes from the thermal conductivity.

- This is the thermal conductivity measured in experiments.

Familiar limits

- When we have (Galilean or Lorentz) boosts, we can fix some of the hydro coefficients.
- Galilean boosts

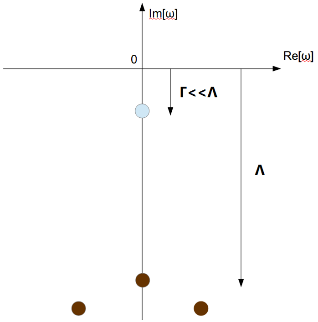
$$\chi_{PP} = mn\,, \quad \rho = ne\,, \quad \sigma_o = \alpha_o = 0$$

$$\pi = mj = mnev$$

- Lorentz boosts

$$\chi_{PP} = \epsilon + p = \mu\rho + Ts\,, \quad \alpha_o = -\frac{\mu}{T}\sigma_o\,, \quad \bar{\kappa}_o = \frac{\mu^2}{T}\sigma_o$$

Introducing weak, long wavelength disorder



- Finite dc conductivities? Relax momentum, ie break translations explicitly. The simplest way to treat disorder perturbatively in a 'mean field' way.

$$\partial_t \pi^i + \partial_j \tau^{ij} = -\Gamma \pi^i + \rho \left(E^i + v_k F^{ki} \right) , \quad \Gamma \ll \Lambda \sim 1/\tau_{th}$$

- The conductivity and associated resistivity become

$$\sigma(\omega) = \sigma_o + \frac{\rho^2}{\chi_{PP}} \frac{1}{\Gamma - i\omega} , \quad \rho_{dc} = \frac{1}{\sigma_{dc}} \sim O(\Gamma) \neq 0$$

Disorder is a (dangerously) irrelevant deformation for the resistivity.

Hydrodynamic signatures in electronic flows

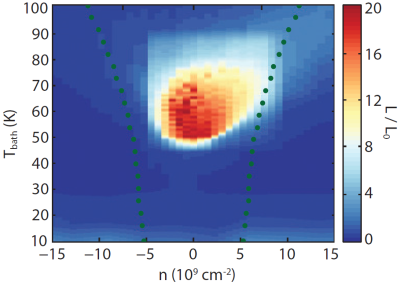
- Wiedemann-Franz law for conventional metals

$$\mathcal{L} = \frac{\kappa_e}{\sigma T} = \frac{\pi^2}{3}\left(\frac{k_B^2}{e}\right)^2 \equiv \mathcal{L}_0$$

The WF law holds because both κ_e, $\sigma \sim \tau_{qp}$.

- In very clean Graphene near the charge neutrality point

$$\kappa_e = \frac{\chi_{PP}}{T\Gamma}, \quad \sigma \sim \sigma_o \quad \Rightarrow \quad \mathcal{L} \sim O\left(\frac{1}{\Gamma}\right) \gg \mathcal{L}_0$$



[Crossno et al, Science 351 6277 (2016)]

Other recent experiments

- Backflows and negative resistance in Graphene due to viscous effects [Levitov & Falkovich, Nat. Phys. 12 (2016)], [Bandurin et al, Science 351 (2016)].
- Viscous contributions to the resistance in restricted channels in $PdCoO_2$ [Moll et al, Science 351 2016].
- Viscous contributions to the resistance, violations of WF law and Hall measurements in WP_2 [Gooth et al, arXiv:1706.05925].
- Dirk van der Marel's talk on Thursday.

Beyond simple hydrodynamics

- How do we compute σ_o, Γ?
- Short-scale disorder?
- Strong disorder?

Need a more microscopic approach!